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Dynamics of a Tick-borne Disease Model with Birth Pulse and Pesticide Pulse at Different Moments*

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Abstract Tick-borne diseases pose a potential risk to public health, which is influenced by the stage structure and seasonal reproduction of tick populations. In this paper, a model that explains the transmission dynamics of pathogens among ticks and hosts is formulated and analyzed, considering birth pulse and pesticide pulse on tick population at different moments. Using the stroboscopic mapping for the disease-free system, we prove a globally asymptotically stable positive periodic solution exists when the pulsed pesticide spraying intensity is less than a critical threshold. Applying the comparison theorem for the impulsive differential system, the conditions for global attraction of the disease-free periodic solution to the investigated system are given. Moreover, we demonstrate the persistence of the studied system and give numerical simulations to verify it. Ultimately, we discuss the case with multiple pesticide sprays and conclude that fewer sprays are more favorable for disease extinction.

Keywords Tick-borne disease, stage structure, double pulse, stability, permanence

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1. Introduction

Tick-borne diseases, like tick-borne encephalitis(TBE) [1], tick-borne relapsing fever [2], Lyme disease [3], have become a major problem for woodland populations living in many parts of Europe, the former Soviet Union and North America. This problem has become increasingly serious over the last 20 years as people spend more time outdoors [4]. Lyme disease is an important infectious disease in the United States, with more than 40,000 cases reported each year, but eight to ten times as many people are actually infected [5]. Lyme disease has a high incidence among forest and field workers [6]. The clinical manifestations include meningitis, encephalitis, neuritis, motor and sensory neuritis and other neurological damage [7]. During the initial phase of the disease, chronic erythema migrans affect the skin, while in

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the intermediate stage, it leads to lesions in the nerves, heart, or joints, and the treatment for nervous system damage has not been effective thus far.

Tick-borne diseases are spread through ticks bites, which are parasitic arthropods that commonly attach themselves to various mammals, including humans. Ticks act as carriers or vectors for several diseases affecting humans and animals [8]. The typical life cycle of ticks spans a duration of 2 to 3 years in their natural habitat, during which they undergo four distinct stages of development: the egg, larva, nymph, and adult phases [6]. Except for the egg stage, each other stage contains three sub-stages: searching for a host, feeding blood and engorgement [9]. The larvae and nymphs are normally parasitic on small rodents (like birds and mice) while the adults mainly feed on medium and large mammals (like sheep and deer) [10].

We can control the prevalence of tick-borne diseases in several ways, such as vaccination [11], biological control [12] and chemical control (pesticide spraying). However, there is currently only one vaccine for TBE, which is not permanent. People need three intramuscular vaccinations (primary series) on day 0, months 1-3 and months 5-12, with a booster shot after 3 years and every 5 years thereafter [13]. Irregular vaccination schedules will result in slightly less effective vaccines [14]. Since most countries do not include tick-borne vaccines in health insurance, the vaccine coverage is low. For biological control, it is impractical to introduce a large number of natural enemies to prey on ticks because of the small number of species of ticks, and it is easy to destroy the original biological chain. Therefore, chemical control i.e., spraying pesticides on animals, grasses and forests to kill ticks, is the most important means of tick control.

At present, a number of modeling efforts have been conducted by many scholars to research tick population dynamics and the spread of tick-borne diseases. Tosato et al. [15] modeled disease transmission in ticks and rodents, and studied the effects of two means of control: insect repellents and acaricides. Analysis suggests that host control with chemical insecticides in areas with rapidly growing tick populations may prolong the duration of the disease and even allow the disease to spread in disease-free areas. Rosà and Pugliese [4] discussed the influence of host population density and structure on the spread of tick-borne diseases, concluding that low host population densities do not sustain cycles of infection, while high host population densities may dilute pathogen transmission. Egyed et al. [16] analyzed the effect of season on Ixodes ricinus activity and the prevalence of infection with major tick-borne disease pathogens in Hungary. Lou and Wu [17] gave basic regeneration numbers of complex disease systems for twelve developmental stages of tick populations.

Recently, a number of researchers have developed epidemiological and population dynamics models with impulsive differential equations [18,19]. Pulsed differential equations exist in almost every field of modern science and have been applied in numerous studies. Wang et al. [20] introduced the impulse control approach to pest biological control by considering the global nature of epidemiological models and bifurcating models with periodic impulse effects. Li et al. [21] constructed a generalized predator-prey system containing a nonlinear pulse to study the effects of pulse-released predators on prey population outbreaks. Tang and Chen [22] presented a stage-structured population system with different fertility function pulses, comparing the dynamic behavior of Ricker functional and Beverton-Holt functional fertility pulses. Sisodiya et al. [23] proposed a model of mosquito-borne disease that considers three means of impulse control at the same moment, namely vac-