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Role of Cooperative Hunting among Predators and Predator-Dependent Prey Refuge Behavior in a Predator-Prey Model*

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Abstract The investigation of predator cooperative hunting and prey refuge is crucial for understanding ecological dynamics. In recent years, the role of cooperative hunting or prey refuge in predator-prey systems has received much attention from researchers. However, the study on the combined effects of predator cooperation and predator-dependent prey refuge in the predation system has not yet been investigated. Therefore, in this paper, we propose a prey-predator model with both the factors of predator-dependent prey refuge and cooperative hunting. The positivity and boundedness of the system's solutions are investigated, followed by the existence and local stability of the equilibrium points. Sufficient conditions for the existence of Hopf bifurcation of the system are obtained. The direction of Hopf bifurcation in the system is investigated by using the center manifold theorem and normal form method. From the analysis of the model, we find that the dependence coefficient m of the prey refuge ratio on the number of predators may be responsible for the stability of the system. The results also indicate that suitable competition coefficients s and cooperative hunting coefficients α between predators may enable the species to coexist in the long run. Furthermore, we observe two limit cycles of the system when the parameters satisfy certain conditions. Finally, the dynamical behavior of the model is performed through intriguing numerical simulations.

Keywords Predator-prey model, hunting cooperation, predator-dependent prey refuge, Hopf bifurcation

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1. Introduction

The predator-prey model is an important mathematical model in population dynamics and an important study branch in the field of biomathematics. Understanding the regulating mechanism in the process of predation and then accurately predicting and estimating the population size of predator and prey is enhanced by studying the

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various features of predator-prey models. When investigating various predator-prey models in recent years, many scholars have taken the cooperative hunting behavior of predator populations into consideration [1–5].

Cooperative hunting means that predator groups work together to improve the efficacy of foraging. For example, cooperative hunting exists in crocodiles [6], lions [7], wolves [8], spiders [9], eagles [10], ants [11] and other species. To more adequately describe the phenomenon of cooperative hunting across predator populations, Cosner et al. [12] proposed a functional response in 1999 in which a predator feeds in a spatially linear pattern and the predators congregate when they encounter a group of prey. In 2010, Berec [13] used ordinary differential equations to simulate a predator-prey model with Holling-II functional response to explain the foraging facilitation between predators and discussed the influence of different intensities of predator interference on the dynamics of the predator-prey model. Following that, in 2017, Alves and Hilker [14] argued that cooperative hunting has an impact on predator population attack rates and that it is necessary to add a cooperative item to predator population attack rates, and proposed the following functional response

$$\Phi(x, y) = (l + \alpha y)x,$$

where x and y denote prey and predator, respectively; both l and α are positive model parameters. l is the attack rate of a predator on prey, α is the cooperative hunting efficiency of a predator, and αy is the cooperative term. Therefore, they developed the following model of cooperative hunting with the Holling-I functional response

$$\begin{cases} \dfrac{dx}{dt} = rx\left(1 - \dfrac{x}{k}\right) - (l + \alpha y)xy, \\ \dfrac{dy}{dt} = e(l + \alpha y)xy - dy, \end{cases} \tag{1.1}$$

where r is the intrinsic growth rate of prey per capita, k is the carrying capacity of prey, e is the conversion efficiency, and d is the predators' per capita mortality rate. All parameters are positive. There have been many results in the discussion of model (1.1). For instance, Zhang et al. [15] investigated the existence and stability of the positive equilibrium point as well as the optimal control problem after introducing the Allee effect into the model (1.1). Pal et al. [16] considered the impact of hunting cooperation and the fear factor on the dynamics of the predator-prey model by incorporating a fear factor into the model (1.1). Halder et al. [17] investigated that cooperative hunting produced both fear and Allee effect under Holling type I and Holling type II functional response. Recently, Thirthar et al. [18] examined the effect of fear in a predator-prey model with additional food, prey refuge, and harvesting by the super predator.

In an ecosystem, fear induced refuge is another fascinating and critical factor that is acquired by prey. It plays an important role in balancing predator-prey interactions. Thus, for an ecosystem, the concept of refuge is worthy of investigation [19]. For modeling purposes, two types of refuges are commonly used by many researchers. The quantity of prey in cover x_r is proportional to prey density x, that is, $x_r = \theta x$. The other one, for which the refuge population is a fixed quantity $x_r = \theta$. Ruxton [20] considered $x_r = \theta xy$, where y is predator density. In the present article, we are interested in investigating the situation where prey in the refuge is related to the number of predators which is $x_r = \theta(1 - \frac{1}{1+my})x = \frac{\theta my}{1+my}x$,