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Numerical Approximation of the Fractional Pine Wilt Disease Model via Taylor Wavelet Collocation Method

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Abstract This article aims to develop a quick and easy Taylor wavelet collocation method with the help of an operational matrix of integration of the Taylor wavelets. Solving epidemiological models ensures the necessary accuracy for relatively small grid points. Finding the appropriate approximations with a new numerical design is challenging. This study examines the fractional Pine wilt disease (PWD) model. Using the Caputo fractional derivative for the fractional order, we developed the novel wavelet scheme known as the Taylor wavelet collocation technique (TWCM) to approximate the PWD model numerically. The results have been compared between the developed method, the Homotopy analysis transform method (HATM), the RK4 method, and the ND solver. The numerical outcomes demonstrate that (TWCM) is incredibly effective and precise for solving the PWD model of fractional order. The approach under consideration is a powerful tool for obtaining numerical solutions to fractional-order nonlinear differential equations. The fractional order differential operator provides a more advanced way to study the dynamic behavior of different complex systems than the integer order differential operator does. The proposed wavelet method suits solutions with sharp edge/ jump discontinuities. Fractional differential equations, delay differential equations, and stiff systems can be solved using this method directly without using any control parameters. For highly nonlinear problems, the TWCM technique yields accurate solutions close to exact solutions by avoiding data rounding and just computing a few terms. Mathematical software Mathematica has been used for numerical computations and implementation.

Keywords Taylor wavelet, Caputo fractional derivative (CFD), system of fractional ordinary differential equations (SFODEs), Pine wilt disease model

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1. Introduction

Mathematical modelling is a potent instrument for studying, investigating, and understanding the spread of various diseases and developing methods to manage them in society. Mathematical modelling makes understanding a disease's spread and defining the essential factors easier. Numerous biological models have been examined from different aspects in this regard. Infectious disease models for humans, animals, plants, and trees have all been developed in the literature. Forests are

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crucial to human life. Its significance is indescribable in words because a single forest can completely rejuvenate the planet. Consequently, it is essential to safeguard trees. In addition to creating a green carpet on Earth, trees give us access to necessities like clean air. Pine wilt disease (PWD) destroys trees within weeks of symptoms showing, despite its majestic and elegant appearance. The pathogens, which include bacteria, viruses, and protozoa, are the leading causes of infectious disorders [1, 2]. The average lifespan of pine trees with PWD infection is a few months. "Wilt diseases" refers to various ailments that affect a plant's vascular system. Plants can be attacked by nematodes, fungi, and bacteria, which instantly destroy them. Viruses can exist in plants as well. Two types of wilt diseases affect woody plants: those that start at the branches and those that begin at the roots. Only a small percentage of infections spread to other plants via the root grafting, typically starting at the stems, where pathogens feed on leaves or bark. Most infections start as lesioning or as pathogens entering the roots directly. The only way to protect the pine forest is to keep it free of disease, as diseased pine trees cannot be spared.

The following set of non-linear equations is used to describe the complex model [3,11,43,44]. $S(\xi)$, $E(\xi)$, $I(\xi)$, $R(\xi)$ and $R(\xi)$ are used to represent the susceptible pine tree class, exposed pine trees, infected pine trees, and susceptible beetle class, respectively. Additionally, $Q(\xi)$ represents the infectious class of beetles.

$$\left.\begin{aligned}
\frac{dS(\xi)}{d\xi} &= \theta - \delta\, S(\xi)\; Q(\xi)\,(1+\epsilon\, Q(\xi)) - \psi\, S(\xi)\,,\\
\frac{dE(\xi)}{d\xi} &= \delta\, S(\xi)\; Q(\xi)\;(1+\epsilon\, Q(\xi)) - (\tau+\psi)\, E(\xi)\,,\\
\frac{dI(\xi)}{d\xi} &= \tau\, E(\xi) - \psi\, I(\xi)\,,\\
\frac{dR(\xi)}{d\xi} &= \lambda - a\, I(\xi)\, R(\xi)\,(1+\kappa\, I(\xi)) - \beta\, R(\xi)\,,\\
\frac{dQ(\xi)}{d\xi} &= a\, I(\xi)\, R(\xi)\,(1+\kappa\, I(\xi)) - \beta\, Q(\xi)\,.
\end{aligned}\right\} \tag{1.1}$$

The symbol θ denotes the entry of new trees into growth. New beetles entering induction are indicated by the symbol λ, and ψ denotes the ratio of dead trees to newly planted ones. Additionally, β reflects the beetle mortality rate, a represents the nematode growth rate, κ denotes the saturation of the beetle infection,ϵ is the infection saturation in trees, δ is the trees contact rate, and τ is the beetle contact rate. The initial conditions are $S(0) = 300,\quad E(0) = 30,\quad I(0) = 20,\quad R(0) = 65$ and $Q(0) = 20$. The parametric values are $\theta = 0.009041, \delta = 0.00166,\quad \epsilon = 0.01,\quad \psi = 0.0000301,\ \tau = 0.002691,\ \lambda = 0.057142,\ a = 0.00305,\quad \kappa = 0.02,\ \beta = 0.01176$.

The fractional mathematical model in Caputo fractional derivative: Now, we incorporate the fractional order into the ODE model. Since fractional calculus has received much attention from researchers lately, various elements of the topic are being considered for investigation. Creating mathematical models based on fractional differential equation and examining their dynamical behaviors are effective and valuable methods of understanding biological issues. The hereditary characteristics, system memory, and non-local distributed effects are all considered by fractional order derivatives and integrals.

Therefore, we modify the system by changing the time derivative with the CFD to present the influence of non-locality. The following fractional SODEs represent