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Nonlinear SEIS Epidemic Dynamics with Fractional-Order Time: Analytical and Numerical Results

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Abstract This study investigates a novel SEIS epidemic model that incorporates fractional-order derivatives to account for the memory effects of the disease spread. The Caputo derivative is specifically employed. Furthermore, the model considers the influence of behavioral changes in susceptible individuals by incorporating a general non-linear function that depends on their population size. Leveraging recent advancements in fractional differential equations theory, we establish the existence of solutions and analyze the critical conditions for the system's steady states to achieve global asymptotic stability. Finally, the validity and applicability of the theoretical model are corroborated through numerical simulations using real-world data on the prevalence of Pneumococcus.

Keywords Non-linear epidemic model, fractional system, stability of equilibria

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1. Introduction

This work deals with the long-term behavior of a nonlinear fractional SEIS epidemic model with recruitment and varying total population size. Indeed, we divide the host population into three interactive compartments denoted by (S), (E), and (I), where $S(t)$ represents the number of susceptible individuals at time t, $E(t)$ is the number of individuals exposed to the infection but not yet infectious, and $I(t)$ is the number of infected individuals. We assume that the susceptible population has a constant recruitment rate A. Furthermore, we model the number of new cases per unit of time by $\beta\varphi\left(S(t)\right)I(t)$ where β is the transmission rate and φ is an increasing function defined on $[0,\infty[$ such that $\varphi(0)=0$. The function φ can be used to describe how different factors affect the rate of infection. For instance, if $\varphi(s)=s$, it means that the contact rate is fixed and does not depend on the number of susceptible individuals. However, $\varphi(s)=\frac{s}{1+ks}$, means that the contact rate declines as more people become aware of the disease, where k is a parameter

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that measures the effect of awareness [1]. The function φ includes in addition other forms of infection rates, such as the square root factor $\sqrt{s}$ [31], which is a special case of the power form S^p that was studied in [22]. We are interested in the long-term effects of the disease outbreak, so we include both the natural death rate μ and the disease-induced death rate ϵ. We also assume that an exposed person becomes infected at the rate α and that an infected person recovers without any disease-acquired immunity, thus becoming susceptible again at the rate λ. Based on these assumptions, the following integer-order SEIS epidemic model is derived:

$$\begin{cases} \frac{dS}{dt}(t) = A - \mu S(t) - \beta\varphi(S(t))\, I(t) + \lambda I(t), \\ \frac{dE}{dt}(t) = -(\mu+\alpha)\, E(t) + \beta\varphi(S(t))\, I(t), \\ \frac{dI}{dt}(t) = -(\mu+\epsilon+\lambda)\, I(t) + \alpha E(t), \end{cases} \tag{1.1}$$

under positive initial conditions $S(0) = S_0$, $I(0) = I_0$, $E(0) = E_0$. In the paper [10], using the geometrical approach of Li and Muldowney, the authors found the threshold for system model (1.1) in the special case $\varphi(s) = s$ which determines whether the disease dies out or persists in an endemic level. The same results for SEIS with vertical and horizontal transmission are established by Korobeinikov using Lyapunov functions [18]. Recently, Naim et al. [25] presented a detailed analysis of a SEIS model with nonlinear force infection. It is shown that if the basic reproduction number is less than one, then the disease-free equilibrium is globally asymptotically stable. Otherwise, a unique positive equilibrium appears and it is locally asymptotically stable. Furthermore, by using the Lyapunov function approach, global asymptotic stability is obtained under additional conditions. Many types of SEIS epidemic models are studied in the literature [2,33,35]. However, to the best of our knowledge, the global stability of system (1.1) is not yet investigated.

On the other hand, many researchers used fractional differential equations to model the evolution of transmissible diseases [20, 29, 30, 36, 37]. These equations involve non-integer order derivatives defined by integrals, making them non-local operators. This feature allows them to capture the memory effect seen in various phenomena. This includes modeling viscoelasticity, polymers, and anomalous diffusion. It also extends to medical applications, such as studying hyperthermia in cancer treatment, and other fields where a non-Markovian approach is more appropriate [5,23,32]. Therefore, fractional derivatives have attracted considerable attention in recent years, as they can be applied to formulate simple and unified models for complex materials and processes. The field of fractional calculus offers various fractional-order derivatives, including Riemann-Liouville, Grünwald-Letnikov, and Caputo derivatives [27]. Among these, the Caputo derivative offers distinct advantages for modeling real-world phenomena. Notably, unlike Riemann-Liouville derivatives, the Caputo derivative of a constant is zero. This simplifies the analysis of fractional differential equations involving the Caputo derivative. Additionally, the Caputo derivative allows for straightforward formulation of initial conditions, similar to classical integer-order differential equations. This eases the process of incorporating real-world data into the model. Finally, the Caputo derivative boasts a well-developed mathematical framework with established results on existence, uniqueness, and stability [27]. This robust foundation allows for confident analysis and reliable conclusions when using the Caputo derivative in models. Due to these advantages, we will employ the Caputo derivative in this work. A formal definition is provided in Appendix A. By integrating system (1.1) and changing its uniform