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Some Local Fractional Hermite-Hadamard-Type and Ostrowski-Type Inequalities for Exponentially s-Preconvex Functions with Generalized Mittag-Leffler Kernel*

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Abstract In this paper, we introduced the concepts of local fractional integral and generalized Mittag-Leffler kernel. Based on these, we establish Hermite-Hadamard integral inequalities and Ostrowski integral inequalities via exponentially s-preconvex functions and s-preconvex functions with generalized Mittag-Leffler kernel.

Keywords Local fractional integral, Mittag-Leffler kernel, exponentially s-preconvex functions, Hermite-Hadamard integral inequalities, Ostrowski integral inequalities

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1. Introduction

In the 19th century, Charles Hermite and Jacques Hadamard demonstrated in references [1, 2] the classical Hermite-Hadamard-type integral inequality of convex functions, which describes an estimate of the integral mean of convex functions:

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)\mathrm{d}x \leq \frac{f(a)+f(b)}{2},$$

where$f : I \subseteq \mathbb{R} \to \mathbb{R}, a, b \in I, a < b$.

In 1938, Ostrowski gave another inequalities that are estimated for the difference between the value of a function and the mean of the integrals in [3], which is called Ostrowski-type integral inequality. Let$f : I \to \mathbb{R}$ be a differentiable function in I^1(the interior of I) and let $a, b \in I, a < b$. If $|f'(x)| \leq L$, for all $x \in [a, b]$, then:

$$\left|f(x) - \frac{1}{b-a}\int_a^b f(x)\mathrm{d}x\right| \leq \frac{L}{b-a}\left[\frac{(x-a)^2-(b-x)^2}{2}\right],$$

where L is the Liphitz constant.

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Inequalities are known to be an important tool for solving many mathematical problems. These two types of inequalities above provide an estimate of the mean of the integrals of a function. They have a very wide range of applications in the field of mathematical and engineering calculations. For further research, most scholars have achieved some innovative results by considering the different convexity of functions, which can be referred to [4–8].

On the other hand, fractional calculus is an important branch of calculus theory. In 1993, Miller et al. established the theory of fractional differential equations on the basis of the fractional calculus operator of Riemann-Liouville. In 2006, Kilbas et al. gave the definitions of Riemann-Liouville left definite integral and right definite integral in [9]:

$$^{\mathrm{RL}}\mathrm{I}^{\alpha}_{a^+}[f(x)] = \frac{1}{\Gamma(\alpha)}\int_a^x (x-\xi)^{\alpha-1} f(\xi)\mathrm{d}\xi, \quad x \geq a,$$

$$^{\mathrm{RL}}\mathrm{I}^{\alpha}_{b^-}[f(x)] = \frac{1}{\Gamma(\alpha)}\int_x^b (\xi-x)^{\alpha-1} f(\xi)\mathrm{d}\xi, \quad x \leq b.$$

Subsequently, many scholars have improved and generalized the definition of fractional calculus operators. For example, Hadamard fractional integral operator, katugampola fractional integral operator, integrated fractional integral operator, ψ-Caputo fractional integral operator, etc.

The phenomenon of fractal is almost everywhere in nature, and the fractal problem is a non-differentiable function problem in mathematics, which is also known as the "mathematical pathological problem" in the world. Fractional calculus can deal with continuous classical power law phenomena, but it cannot solve discontinuities. Therefore, focusing on the mathematical-pathological problems that Newton-Leibniz calculus cannot handle, Yang proposed local fractional derivatives and local fractional integrals of non-differentiable functions in references [10, 11], which is also known as Yang's fractal theory. At present, Yang's fractal theory has been widely used in the field of engineering mechanics and differential equation calculation, which can be referred to in [12–17]. Based on the theory of local fractional calculus, the study of integral inequality has also achieved new results. For example, according to Yang's fractal theory, Sun proposed a local fractional integral operator with Mittag-Leffler kernel in [18], and obtained some inequalities about the h-preconvex function. Subsequently, Sun and Xu et al. used them to study the Hermite-Hadamard local fractional integral inequality [19, 20] . In 2024, Sun studied the local fractional integral inequalities of the Hermite-Hadamard type and Ostrowski type of the generalized h-preconvex function in [21].Wei gave a new fractal modeling for the nerve impulses based on local fractional derivative in [22].More recent results can be found in References [23–25].

Therefore, inspired by the existing results, this paper will construct some local fractional Hermite-Hadamard-type and Ostrowski-type inequalities for exponentially s-preconvex functions with generalized Mittag-Leffler kernel on the Yang's fractal sets. By taking some specific values for the parameters in the main results, it is possible to obtain some known results or new conclusions in the references.