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Results on Interval-Valued Optimization Problems for Vector Variational-like Inequalities

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Abstract The objective of this article is to present a new class of vector interval-valued variational-like inequality problems. Based on the concepts of LU optimal and weakly LU optimal solutions, we derive some relations between the interval-valued programming and variational-like inequality problems. The study of the interplay between interval-valued optimization problems (IVOP) and vector variational-like inequalities (VVLI) combines theoretical advancements under the concept of differentiability and μ- invexity. Furthermore, to demonstrate the established linkages, we provide examples and an example demonstrates how well the vector variational inequality problems may be applied to deal with (MOIVP) problems.

Keywords Interval-valued programming, invexity, LU-optimal, variational-like inequality problem

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1. Introduction

In 1980, Giannessi [11] initially introduced vector variational inequality (VVI) in Euclidean space via finite dimensions. The relationship between variational-like inequality and certain mathematical programming problems was identified by Parida et al. [25], who also offered a theory for the existence of a solution to variational-like inequalities. Later on Deng [9] enunciated necessary and sufficient conditions for the existence of weak minima in constrained convex vector optimization problems. New iterative techniques that, under certain conditions, can be used to solve mixed variational-like inequalities were studied by Noor et al. [22] using convergence analysis. In Variational Inequalities, along with their spectrum of applications, Kinderlehrer et al. [17] and Mordukhovich [20] introduced an applicative approach along with exciting emerging fields like medicine, finance, optimization, system stability, environmental, science and phase transformations (also see [34]).

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Oveisiha et al. [23] considered two generalized minty vector variational like inequalities and investigated the relations between their solutions and vector optimization problems for non-differentiable α-invex functions. In general mathematical programming coefficients are always considered as deterministic value, but in reality it observed that the parameters may not be known certainly. Numerous techniques, such as fuzzy numbers and stochastic processes, can be used to manage uncertainty in the real world. Still, given the complexity and incompleteness of the data, it can be challenging to identify a suitable probability distribution or membership function. Because of this, there has been a lot of interest in the uncertain optimization problem recently. An approach to overcome the uncertain optimization problem where only the range of the coefficients is known, is interval-valued programming and it does not require the assumption of probabilistic (stochastic programming) and possibility distributions (fuzzy programming).

As an extension of convexity, Zhang et al. [36] explored LU-convexity to figure out the optimality criteria for real-valued maps with the consideration for optimal solutions with interval values. The role of invexity in variational-like inequalities is the same as that of classical convexity in variational inequalities; this indicates that variational-like inequalities are well-defined in terms of invexity. It has been noted that all the results for variational-like inequalities are derived in the setting of classical convexity, (see Gupta et al. [12], and Jennane et al. [15]). Subsequently, numerous analogous inequalities were obtained for various categories of preinvex functions; (refer to Noor et al. [21]).

Very recently, the equivalence between generalized Stampacchia vector variational inequality and quasi LU-efficient solutions to interval-valued vector optimization challenges was established through Upadhyay et al. [30], who additionally defined a generalized LU-approximately convex function. In the meantime, Laha et al. [18] linked multi-objective optimization problems and vector variational-like inequalities in this chain. Since optimization algorithms usually offer only approximate solutions and terminate in a finite number of steps in a wide range of real-world problems, Evtushenko [10] provided an association between Farkas' theorem and linear and quadratic programming. It has recently been established that vector variational inequalities and vector optimization problems are related to convexificators by Bhardwaj et al., [4] khan et al. [16], Pany et al. [24], and Upadhyay et al. [31] .

Over the past few years, various extensions and generalizations of the variational inequalities have been introduced. A number of outstanding reviews, including those by Chang et al. [6] , Mohapatra et al. [19], have been published to provide an overview of the state of knowledge on the variational-like inequalities. In the recent past, Antczak et al. [2,3], Abdulaleem N. [1], Huy et al. [14], and Treanţă [29] have shown that vector variational inequalities can describe the optimality conditions for vector optimization problems under some certain conditions. Recently, many researchers have opened up a new dimension of best proximity point (BPP) results for an applicative approach to optimization. For this, Younis et al. [33] invoked the BPP for multivalued mappings with the application of the equation of motion. For further applications in this set up, one can see the noteworthy work done in Dar et al. [8].

In this paper, by utilizing the concepts of LU optimal and weakly LU optimal solutions some relations are investigated between the interval-valued optimization and variational-like inequality problems. Moreover, examples are given to validate