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Optimal Control of a Delayed Spatiotemporal Epidemic Model

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Abstract This study presents an advanced delayed spatiotemporal epidemiological model that incorporates a Holling type II incidence rate to capture the saturation effects observed in disease transmission dynamics during the COVID-19 pandemic. The model integrates two crucial intervention measures - vaccination of susceptible individuals and hospitalization of severe cases - while accounting for both spatial diffusion and the latent period within the epidemic compartments. This framework facilitates the precise optimization of vaccination and hospitalization strategies as functions of spatial location and temporal evolution, yielding new insights into spatially targeted public health interventions. We rigorously analyze the model equilibrium points, establishing conditions for their existence and local stability. An optimal control problem is formulated, uniquely considering the combined effects of spatial diffusion and latent period, with controls dynamically varying across space and time. The well-posedness of the control problem is verified, supported by proofs of existence, uniqueness, positivity, and boundedness of the strong solution. First-order necessary optimality conditions are derived, characterizing the optimal vaccination and hospitalization strategies through state and adjoint variables. Numerical simulations across diverse intervention scenarios demonstrate the effectiveness of adaptive, space-time-specific control strategies in mitigating COVID-19 transmission. This work offers a novel mathematical and computational approach to the optimal spatiotemporal management of epidemic control measures.

Keywords Delayed spatiotemporal epidemic model, vaccination, hospitalization, reaction-diffusion equations, optimal control

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1. Introduction

With the emergence of epidemics like SARS, Ebola, and COVID-19, which are detrimental to individual health and societal stability, there is an increased need for policymakers and researchers to understand the patterns, the behaviors, and the dynamics of the diseases in order to prevent and control their spread. Mathematicians and immunologists collaborate to create models that can predict the course

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of an epidemic. Classical compartmental models for epidemics use variables to describe the state of an infectious disease's exposed subpopulation. Each parameter incorporated represents a fundamental factor, such as the rate of transmission of the infectious agent, the mortality rate, and other data. It is possible to build very reliable models that will allow determining the best treatments as well as the respective impacts of the factors that influence this disease with a thorough knowledge not only of applied mathematics but also of the biology of the disease. These models are determined by the transmission method, the nature of the disease, its curability, and the body's ability to develop immunity after recovery, etc. [15, 18]. Following the 1905 - 1906 Bombay plague epidemic, the year 1927 witnessed the emergence of the SIR model, which is considered one of the first and most used compartmental models [20]. This model assumes that the population is divided into three subpopulations: susceptibles, infected, and recovered. Later, several compartmental models were inspired by the SIR and have been widely applied to research infectious disease outbreaks [10, 11, 18, 36, 44] and to examine potential policy responses. Since the COVID-19 outbreak, many authors have examined the dynamics of the disease's spread in light of various situations [14, 32, 35]. In particular, Ndaïrou et al. [32] presented a compartmental model of COVID-19 transmission dynamics with a case study of Wuhan where they focused on the transmissibility of superspreaders individuals. Samui et al. [35] proposed a compartmental mathematical model to predict and control the transmission dynamics of COVID-19 pandemic in India. They performed local and global stability analysis for the infection free equilibrium point and the endemic equilibrium point with respect to the basic reproduction number. Diagne et al. [14] formulated and analyzed a mathematical model of COVID-19 transmission incorporating two key therapeutic measures: vaccination of susceptible individuals and treatment of infected individuals. In their model, they included a compartment (E) for exposed persons, responsible for the incubation period. For a model in which the size of the problem is relevant, It is preferable to attain the same dynamics with fewer compartments for a model. The dynamics won't, however, be exactly the same if a compartment (like E) is removed. In fact, we think that the formulation of the delay equation could better capture the effect of "delay" brought on by the introduction of new measures (as was the case with the COVID-19 pandemic), where there is a delay of several days between the introduction of a new public health order and when its effects start to be noticed. Additionally, delays may vary according to the stage of the epidemic, resulting in state-dependent delays. Although we won't look at such a case here, it is important to first grasp the case of constant delay, which is what this present effort is trying to do. Furthermore, from a mathematical perspective, such a formulation is intriguing. Several authors have reflected on this matter by studying the existence of solutions and bifurcations of time-delayed compartmental models [7, 19, 24, 27]. Furthermore, many studies seek the most effective strategy for reducing infection rates while minimizing implementation costs [9, 25]. However, all these works didn't take into consideration the spatial diffusion that is crucial to the propagation of epidemics and must be taken into account when implementing control strategies (for instance, an area that containts more infected individuals needs more attention). As a result, some authors [2–5, 16, 21, 22] investigated spatiotemporal models in which the disease spread was represented as a system of reaction-diffusion equations. However, to our knowledge, no deterministic model has treated an optimal control problem that takes into consideration simultaneously the spatial diffusion