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The Fractal-Fractional Mathematical Model Analysis of the Impact of HIV/AIDS on the Working-Class in Ethiopia

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Abstract The HIV/AIDS epidemic profoundly affects the working-class population, resulting in increased mortality and morbidity rates and causing substantial labour loss across various sectors. To assess the epidemic's impact on the specified class, a fractal-fractional-order mathematical model was formulated based on Atangana-Baleanu-Caputo operator. This model encompasses seven compartments, and the existence and uniqueness of its solutions were verified based on the Banach fixed-point theorem, contraction mapping concepts, and Hyers-Ulam stability criteria. The mathematical model was analyzed to understand the effects of HIV/AIDS on the working-class population. Real data from Ethiopia were utilized to validate the model. Subsequently, the model was extended to optimal control fractal-fractional models, incorporating different control strategies. Numerical simulations were performed using MATLAB R2019a to support the analytical solutions. The study's results demonstrated that the fractal-fractional-order model provides a comprehensive understanding of the complexities of HIV/AIDS infection. The findings suggested that increasing the number of infected productive members of the population can help to control the spread of the disease by reducing the inequality caused by HIV/AIDS. Furthermore, the numerical simulation of the optimal control model indicated that decreasing non-productivity reduces the number of infected individuals. Therefore, effective management of the infection holds the potential to eradicate the disease in the country.

Keywords Hyers-Ulam stability, HIV/AIDS, working-class population, optimal-control, fractional order, fractal-fractional

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1. Introduction

The human immunodeficiency virus (HIV) affects approximately 39.9 million individuals worldwide, and the sub-Saharan African region bears a disproportionate share of this disease burden. Ethiopia, a sub-Saharan nation, grapples with this HIV/AIDS crisis Kanki et al. [1]. Developing robust behavioral change and educational initiatives at the community level is pivotal for mitigating the prevalence of

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prejudice and stigma against people living with HIV/AIDS (PLWHA) Arefaynie et al. [2]. The human immunodeficiency virus (HIV) undermines the immune system, leading to acquired immunodeficiency syndrome (AIDS), which can be transmitted through unprotected sexual intercourse, shared drug injection equipment, or from mother to child during childbirth or breastfeeding. Preventive strategies encompass consistent condom usage, routine testing and counseling, timely initiation and follow-up of antiretroviral therapy (ART), pre-exposure prophylaxis (PrEP), and measures to avert mother-to-child transmission [3, 4].

The HIV/AIDS crisis intersects with COVID-19 and exacerbates numerous problems for those living with and infected by the disease. People living with HIV/AIDS demand a comprehensive global response to this pandemic. Troubling new data from the UNAIDS global AIDS update of 2023 portend a grim outlook, as progress has stagnated, resources have dwindled, and inequities have intensified. The global population of individuals living with HIV increased from 38 million in 2020 to 38.4 million in 2021 during the COVID-19 pandemic and 39.9 million in 2023, with the African region accounting for 67% of these cases. HIV-affected populations have also grown in the Middle East and North Africa. Maintaining the current trajectory will result in millions of additional HIV infections and AIDS-related deaths [5]. Therefore, further investigation and research on HIV/AIDS are imperative to re-examine the nature of the pandemic and its effects on productive forces and other community groups. If left uncontrolled, HIV/AIDS infection will continue to pose a grave public health and economic challenge worldwide [6, 7]. Global efforts must focus on raising awareness, eliminating stigma, and addressing the socioeconomic factors that contribute to the endemic nature of this crisis. Ongoing research and public health initiatives are essential in the fight against HIV/AIDS Shiferaw et al. [3]. The lack of comprehensive data on behavioral trends in Ethiopia hinders the accurate interpretation of HIV prevalence and incidence, obscuring a clear understanding of the current state of the HIV/AIDS epidemic in the country Adal [8].

Fractional calculus encompasses the study of non-integer order derivatives [9, 10]. It is employed to describe a typical diffusion process that can influence our understanding of disease transmission in unconventional ways. This domain of research is relatively well-developed in fields such as physics and engineering, and its application to infectious disease modeling remains an emerging area of inquiry. It is crucial to note that the utilization of mathematical models for infectious diseases with a fractal-fractional order model is employed to capture the intricacy and self-similarity of disease transmission models. The spatial distribution of infections, or temporal patterns of epidemic waves, can exhibit fractal-like characteristics Abu and Saadeh [11]. This approach is valuable for capturing the heterogeneity and irregularity of real systems, providing a more comprehensive understanding of the spatial and temporal dynamics of diseases. The Atangana-Baleanu operator of fractional calculus in the sense of Caputo is a specialized operator of fractional calculus introduced to extend the classical Caputo fractional operator Atangana [12]. Fractional operators are mathematical tools that generalize the concept of derivatives and integrals to non-integer orders, enabling a more precise description of various phenomena, particularly those related to memory and distance dependence [10, 13–16]. Atangana and Baleanu introduced this fractional operator in their work on fractional calculus and integral transformations of generalized functions to provide an alternative approach to fractional calculus that incorporates the Caputo meaning and considers certain related restrictions. The Atangana-Baleanu operator