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Local Existence for the Generalized Navier-Stokes-Maxwell Equations*

Xinru Cheng[1], Liangbing Jin[1,†] and Rong Zou[2]

Abstract In this paper, we establish the local existence for the generalized Navier-Stokes-Maxwell system with the fractional velocity dissipative term $\Lambda^{2\alpha}u$ and fractional magnetic dissipative term $\Lambda^{2\beta}B$. Moreover, we establish the global existence of strong solutions to this generalized model.

Keywords Navier-Stokes-Maxwell system, fractional dissipative, local existence, global existence

MSC(2010) 35Q30, 35Q35, 76A03, 76Q05.

1. Introduction

In this paper, we consider the following Cauchy problem for the Navier-Stokes-Maxwell system:

$$\begin{cases} u_t + u\cdot\nabla u + \nabla P + \Lambda^{2\alpha}u = j\times B, \\ E_t - \mathrm{curl}B = -j, \\ B_t + \mathrm{curl}E + \Lambda^{2\beta}B = 0, \\ \mathrm{div}u = 0, \mathrm{div}B = 0, \\ u(x,0) = u_0(x), B(x,0) = B_0(x), E(x,0) = E_0(x), \end{cases} \tag{1.1}$$

where $x\in\mathbb{R}^3$ and $t>0$. $u=u(x,t), B=B(x,t)$ and $E=E(x,t)$ denote the velocity field, the electric field and the magnetic field of the fluid, respectively. P denotes the scalar pressure and j denotes the electric current density which is given by Ohm's law. Moreover,

$$j = \sigma(E + u\times B), \tag{1.2}$$

where $\sigma>0$ denotes the electric resistivity. For simplicity, we set $\sigma=1$. The fractional Laplacian operator $\Lambda^\alpha = (-\Delta)^{\alpha/2}$ is defined through the Fourier transform

$$\widehat{\Lambda^\alpha f}(\xi) = |\xi|^\alpha \hat{f}(\xi),$$

[†]the corresponding author.
Email address:chengxr@zjnu.edu.cn,lbjin@zjnu.edu.cn,rzou@hpu.edu
[1]School of Mathematical Sciences, Zhejiang Normal University, Jinhua, Zhejiang 321004, China
[2]Department of Mathematics, Hawaii Pacific University, Honolulu 96813, Hawaii, USA
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where the Fourier transform is given by

$$\hat{f}(\xi) = \int_{\mathbb{R}^3} e^{-ix\cdot\xi} f(x)\, dx.$$

When $\alpha = 1,\ \beta = 0$, the equations (1.1) are reduced to the classical Navier-Stokes-Maxwell equations. Masmoudi [1] proved the global existence and uniqueness of strong solutions if the initial data $u_0,\ E_0,\ B_0 \in L^2(\mathbb{R}^2) \times H^s(\mathbb{R}^2) \times H^s(\mathbb{R}^2)$. Using energy estimates and Brezis-Gallouet inequality, Kang and Lee [2] reproved the global existence of regular solutions to the 2D system and obtained a blow-up criterion. In [3], by using Fujita-Kato's method, Masmoudi and Yoneda proved the local existence of the solutions and the loss of smoothness for three dimensional large periodic initial data. Ibrahim and Keraani [4] showed the global existence of the strong solution provided that the initial data $\|u_0\|_{\dot{B}^{1/2}_{2,1}} + \|E_0\|_{\dot{H}^{1/2}} + \|B_0\|_{\dot{H}^{1/2}}$ is small enough. Germain et al. [5] simplified the proof in [4] and lowed the regularity of the initial velocity field in the $\dot{H}^{1/2}(\mathbb{R}^3)$ by using $L^2_t(L^\infty_x)$ estimate on the velocity field. Arsenio and Isabelle [6] proved that global solutions exist under the assumption that the initial velocity and electromagnetic fields have finite energy, and the initial electromagnetic field is small in $\dot{H}^s(\mathbb{R}^n)$ with $s \in [\frac{1}{2}, \frac{3}{2})$. As for the generalized Navier-Stokes-Maxwell system, Jiang [7] proved the global existence and uniqueness of strong solution when $\alpha \geq \frac{3}{2},\ \beta = 0$. In addition, there are many regularity criteria results for the equations (1.1) in [8–11].

Now we state our main theorems as follows:

Theorem 1.1. *Assume the initial data $u_0, E_0, B_0 \in H^s(\mathbb{R}^3)$ satisfying $\mathrm{div}u_0 = \mathrm{div}B_0 = 0$ for $s > \max\left\{\frac{5}{2} - 2\alpha, \frac{3}{2} - \alpha, \frac{3}{2} - \beta, 0\right\}$, then there exists a time $T_* > 0$ such that the equations (1.1) have a unique solution (u, E, B) with*

$$u \in L^\infty(0, T_*; H^s(\mathbb{R}^3)) \cap L^2(0, T_*;\ H^{s+\alpha}(\mathbb{R}^3));$$

$$B \in L^\infty(0, T_*; H^s(\mathbb{R}^3)) \cap L^2(0, T_*;\ H^{s+\beta}(\mathbb{R}^3));$$

$$E \in L^\infty(0, T_*; H^s(\mathbb{R}^3)).$$

Moreover, we could obtain $u,\ E,\ B \in C_w([0, T_*];\ H^s(\mathbb{R}^3))$.

Theorem 1.2. *Assume $\alpha \geq \frac{5}{4}$, $\beta \geq \frac{7}{4}$, $u_0, E_0, B_0 \in H^s(\mathbb{R}^3)$, $s > \max\left\{\frac{5}{2} - 2\alpha, \frac{3}{2} - \alpha, \frac{3}{2} - \beta, 0\right\}$ with $\mathrm{div}u_0 = \mathrm{div}B_0 = 0$, then the Navier-Stokes-Maxwell system (1.1) has a global classical solution.*

The organization of this paper is presented as follows. Firstly, we introduce some lemmas in Section 2. Secondly, we construct the approximate solutions and prove the local wellposedness in Section 3 by using Fourier truncation method. Finally, in Section 4, we justify the global existence to the system (1.1).

2. Preliminaries

In this section, we recall some elementary lemmas which will be used in our proof.

Lemma 2.1. *[12] Define the Fourier truncation S_R as follows:*

$$\widehat{S_R f}(\xi) = 1_{B_R(\xi)} \hat{f}(\xi) = \begin{cases} \hat{f}(\xi), & |\xi| \leq R, \\ 0, & |\xi| > R, \end{cases}$$