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Existence and Simulation of Solutions for a Class of Fractional Differential Systems with *p*-Laplacian Operators on Star Graphs*

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Abstract This paper is mainly concerned with the existence and simulation of solutions for a class of Caputo fractional differential systems with p-Laplacian operators on star graphs. The Hyers-Ulam stability of the systems on star graphs is also proved. Furthermore, an example on a formaldehyde graph is presented to demonstrate the practicality of the main results. The innovation of this paper lies in combining a fractional differential system with a formaldehyde graph, interpreting the chemical bonds as the edges of the graph, and exploring the existence and numerical simulation of solutions to the fractional differential system on this unique graph structure.

Keywords Fractional differential systems, p-Laplacian operator, star graphs, existence, Hyers-Ulam stability

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1. Introduction

Fractional differential equation is a generalization of integer-order differential equation, which can describe some complex phenomena in nature and engineering more accurately. For example, fractional differential equations provide a more appropriate model for describing diffusion processes, wave phenomena and memory effects [2,4,5,11,19,21]. In addition, fractional differential equations also show advantages in dealing with singular systems and nonlinear problems. Therefore, fractional differential equations are widely used in many fields, including physics, biomedicine and engineering [1, 6–8, 12, 16, 28]. For example, Dang [7] proposed a new fractional order model to describe the mechanical behavior of viscoelastic materials with memory effects. Abdullaeva [1] introduced a new fractional model on lithiumion batteries and discussed the application of fractional differential equations in engineering.

Graph theory is a branch of mathematics that mainly studies networks formed by the interconnection of nodes through edges. It originated in 1736 when Euler

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published a paper on “the Seven Bridges of Konigsberg Problem”. Since then, graph theory has become widely applied in sociology, traffic management, telecommunications and other fields [3,23,24].

A star graph is a special graphical structure in which one central node is connected to multiple peripheral nodes without any connections between them. In [17], Mehandiratta et al. has established the sketch of the directed star graphs. The star graph consists of the vertex set $\{\nu_0, \nu_1, ..., \nu_k\}$ and the edge set $\{\overrightarrow{\nu_i\nu_0}, i = 1, 2, ..., k\}$, where $l_i = |\overrightarrow{\nu_i\nu_0}|$ and ν_0 is the joint point.

As is well known, differential equations on star graphs have profound application backgrounds, which can be applied to different fields, such as chemistry, bioengineering and so on. For instance, in the chemical molecular structure, each solution function μ_i on any edge means bond strength, bond energy and bond polarity. The origin of fractional differential equations on star graphs dates back to the research of Graef et al. [10]. They first studied fractional differential equations on a star graph with two edges and proved the existence result by the fixed point theorem. Mehandiratta et al. [17] extended the 2-sided star graph studied by Graef et al. [10] to a k-sided star graph, and converted the equation into an equivalent fractional differential system defined on $[0, 1]$ by transformation $t = \frac{x}{l_i} \in [0, 1]$. The uniqueness result was proved through the use of Banach's contraction principle. Zhang et al. [27] added a function $\lambda_i(x)$ on the basis of the reference [17], and proved the existence result by the fixed point theorem. In addition, Su et al. [18] discussed the existence of a coupled fractional differential system on a glucose graph and proved the Hyers-Ulam stability of solutions to the system. It can be seen that in literatures [10,17,27], attention was mainly focused on the existence of solutions to the fractional differential systems, while Su et al. [18,22,25,26] conducted numerical simulations without combining traditional star graphs.

Inspired by the above references [10,15,17,18,22,27], we study the existence and Hyers-Ulam stability of the solution to the boundary value problem with p-Laplacian operator on star graphs as follows

$$\begin{cases} \phi_p\left({}^cD^{\eta}_{0+}\mu_i(x)\right) = -\lambda_i(x)\hbar_i\big(x, \mu_i(x), {}^cD^{\theta}_{0+}\mu_i(x)\big), \\ \mu_i(x)|_{x=0} = 0, \ i = 1, 2, ..., k, \\ \mu_i(x)|_{x=l_i} = \mu_j(t)|_{x=l_j}, \ i, j = 1, 2, ..., k, \ i \neq j, \\ \sum\limits_{i=1}^{k} \mu_i'(x)|_{x=l_i} = 0, \end{cases} \tag{1.1}$$

where ${}^cD^{\eta}_{0+}, {}^cD^{\theta}_{0+}$ are both Caputo fractional derivative operators, $\eta \in (1, 2]$, $\theta \in (0, 1]$, $p \in (1, 2)$, $\lambda_i(x) \neq 0$, $\lambda_i \in C[0, 1]$, $\hbar_i \in C([0, l_i] \times \mathbb{R} \times \mathbb{R})$ and $\phi_p(s) = sgn(s) \cdot |s|^{p-1}$. The existence and Hyers-Ulam stability of the solutions to system (1.1) are discussed. Moreover, the approximate graphs of the solution are obtained. The innovation of this paper lies in combining a fractional differential system with a formaldehyde graph, interpreting the chemical bonds as the edges of the formaldehyde graph, and exploring the existence and numerical simulation of solutions to the fractional differential system on this unique graph structure.

The outline of the paper is organized as follows. In Section 2, some basic definitions and lemmas are presented. In Section 3, the existence of the solution to the Caputo fractional derivative system is obtained by the Banach and the Krasnoselakii fixed point theorems. In Section 4, the Hyers-Ulam stability of the system