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A Comparative Analysis Between Adomian Decomposition Method and Differential Transformation Method for Solving Some Second-Order Ordinary Differential Equations

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Abstract The differential transformation method (DTM) and Adomian decomposition method (ADM) are two numerical methods that can be used to solve various differential equations. In this work, we compare the accuracy, convergence, and computational complexity of these two methods by using them to solve second-order nonlinear equations using a new differential operator for the second-order equation. We also used both methods to solve second-order nonlinear differential equations.

Keywords Differential transformation method, Adomian decomposition method, second-order ordinary differential equations

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1. Introduction

Quantitative descriptions of many models in the physical, biological, and even social sciences are provided through the use of differential equations. These descriptions are usually made in terms of unknown functions of one, two, or more independent variables and relationships between the derivatives of these variables. If two or more independent variables are involved, the differential equation is called partial differential equation(PDE). Otherwise, it is called an ordinary ordinary differential equation(ODE) [28] . Modeling using differential equations is crucial as it provides relevant insights into the dynamics of many engineering and technical equipment and processes [20, 22]. However, many such models involve differential equations that are inherently nonlinear and difficult to solve. Many numerical methods have been developed to solve various differential equations that cannot be solved analytically [3]. But most numerical methods require discrimination, militarization. The rapid advancement of technology in today's ear has created an increasing need for scientific computing when processing and analyzing big date embodied in large amounts of real-life modeling phenomena. Numerical methods for solving nonlinear

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(ODEs) and (PDEs) are at the heart of many scientific calculations [21,30]. Differential equations have become a useful tool for describing these natural phenomena in scientific and engineering models. Therefore, it becomes important to be familiar with all traditional and recently developed methods for solving differential equations and their implementation. Although many standard methods exist for solving differential equations, more efficient methods still need to be developed or investigated [14]. In 2023 H. Chen and others studied a two-grid temporal second-order scheme for the two-dimensional nonlinear Voltmeter integro-differential equation with weakly singular kernel [13]. This year a study was conducted a computational technique for computing second-type mixed integral equations with singular kernels by A. M. S. Mahdy et al [18]. While AMR. Mahdy and others solved the fractional integro-differential equations using least squares and shifted Legender methods [19]. The main question of this research is which is better, the ADM or the DTM for solving second-order equations?

2. Adomian method

The Adomian decomposition method demonstrates rapid convergence of the solution and provides several significant advantages. This method was introduced and developed by George Adomian form the 1970s to the 1990s [2], as noted by Wen Jin and Yani [17].

Advantages of ADM: It is easy to understand and can be used ot solve many types of linear and nonlinear systems, such as algebraic equations, ordinary and partial differential equations, linear and nonlinear integral equations, differential equations, integral nonlinear stochastic operator equations, etc [10].

3. Analysis of Adomian decomposition method

From [23], we consider of second-order ordinary differential equations with constant coefficients of the form,

$$u'' + (m-2n)u' - n(m-n)u = q(x,u), \tag{3.1}$$
$$u(0) = A, u'(0) = B,$$

where $q(x,u)$ is a nonlinear function, A, B, n, m are constants. The differential operator, is as follows,

$$L(.) = e^{nx}\frac{d}{dx}e^{-mx}\frac{d}{dx}e^{(m-n)x}(.). \tag{3.2}$$

The inverse operator L^{-1} is therefore considered a quadratic integral operator as demonstrated below,

$$L^{-1}(.) = e^{-(m-n)x}\int_0^x e^{mx}\int_0^x e^{-nx}(.). \tag{3.3}$$

The Adomian decomposition method introduces the solution $u(x)$ and the nonlinear function $q(x,u)$ as infinite series,

$$u(x) = \sum_{n=0}^{\infty} u_n(x), \tag{3.4}$$