

A Caputo-Based Fractional Order Modelling of COVID-19 in Nigeria

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Abstract Our focus in this work is the proposition of a fractional order model based on Caputo fractional derivatives for the understanding of how coronavirus disease is transmitted in a community, using Nigeria as a case study. By using Laplace transform, we show that the state variables of the model are non-negative at all times and show the existence and uniqueness of solutions for the model. Thorough analysis of the model shows that the model is Ulam-Hyers-Rassias stable and that its disease-free equilibrium is locally and globally asymptotically stable whenever the reproduction number of the disease is less than unity. By gathering real-life data about the disease in Nigeria from accredited authority, Nigerian Centre for Disease Control (NCDC), we estimate parameters driving the spread of the disease by fitting this data to our model. By adopting these parameter estimates, using MATLAB, we perform the numerical simulation of the model with a view to validating results from qualitative analysis of the model. Numerical results show that plots for the model at different fractional orders have major determining influence on various compartments of the model as it varies. Various distinct results were observed for each of the compartments in different fractional orders, highlighting the importance of consideration of the fractional order in modelling the highly contagious COVID-19 disease. This work highlights the advantage of fractional order model over the classical integer order model in the sense that the solution obtained for the fractional order model possesses a higher degree of freedom that enables variation of the system so as to obtain as many preferable responses of the different classes as desired since variation of fractional order ξ can be done at any preferable fractional rate 0.7, 0.4, 0.2 etc.

Keywords Fractional order, Caputo derivatives, Laplace transform

MSC(2010) 26A33, 34A08, 34D20, 34D23, 92B05.

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1. Introduction

The origin of COVID-19 which is a deadly, highly contagious disease caused by Severe Acute Respiratory Syndrome coronavirus 2 (SARS-COV-2) is Wuhan city of China [1]. It has its mode of transmission from human to human through direct contact with infected persons and surfaces that are contaminated droplets from infected individuals [2]. It is revealed through clinical evidences that COVID-19 has its incubation period of 2 to 14 days [3–5, 8], the period during which infected individuals with the disease start showing clinical symptoms of the highly communicable disease. The infected individuals with the disease during this period of incubation may or may not show symptoms of the disease; regrettably, they are capable of transmitting the disease to other individuals that they come in contact with. Coughing, breathing difficulties, and fever are the symptoms of the disease [9]. It is reported that as of November 6th, 2022, with 6.5 million reported deaths globally [4, 10]. For now, the deadly disease do not have a clinically proven drug to combat its spread, though there are lots of control measures to combat the spread of the disease, such as regular usage of nose masks while in public, regular hand washing with soap or sanitizers and observance of social distances while in public space [5, 11, 12]. The good news is that there is availability of clinically proven vaccines to combat the spread of the disease, which include: the mRNA-1273 Moderna vaccine, BNT162b2 Pfizer-BioNtech vaccine and a highly efficacious vaccine against different variants of CPVID-19 developed in the United States, Johnson and Johnson vaccine [6, 7].

As soon as the COVID-19 pandemic started ravaging the whole world, mathematical epidemiologists went to their studying table to complement the works of the healthcare providers and policy makers in health sectors to combat the spread of the deadly disease by developing several mathematical models including the ones in [10, 11, 13–18, 20–26]. In the work of Gumel et al. [16], a primer for COVID-19 was established. Their work was used to have an understanding of how the disease was transmitted in the early stage of the pandemic in the USA. Okunoghae and Oname [18], in the works on modelling of the transmission dynamics of disease in Nigeria, they incorporated three COVID-19 safety protocols: use of face masks while in public, regular hand washing with soap and hand sanitizers and maintenance of social distance while in public into their model and obtain thresholds for the percentage of the usage of these safety protocols to ensure that the affliction arising from the pandemic is mitigated in the community. Sowole et al. [25] in their work explored and incorporated a linear regression method into the model they proposed and used this to make forecast about the early stage of the disease. Oname et al. [20] in their work proposed a model formulated using fractional calculus with fractional order to study the spread of the co-infection of COVID-19 and Hepatitis B virus, using real-life data from Wuhan city of China. They illustrated that transmission rate for each of the two diseases can have a great impact on the dynamics of the co-infection of the two diseases and to have adequate and effective control of the interaction between the two disease in the population under reference, there is need for concerted efforts to be exerted towards the prevention of infection of either or both diseases.

Due to limitation of classical integer order derivatives, its inability to capture the memory effect, modelling of communicable diseases using fractional order derivatives incorporating fractional differential operators with its merit being that it can capture memory effects which is the major motivation for the development of such