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Numerical Approximation of the System of Fractional Differential Equations Using the Fibonacci Wavelet Collocation Method

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Abstract This study uses the Fibonacci wavelet collocation method (FWCM) to solve the system of fractional differential equations. Here, we introduce the innovative Fibonacci wavelet method with the help of an operational matrix of integration generated by the Fibonacci polynomials to compute the approximate solution of the linear and nonlinear fractional differential equations. The Fibonacci wavelet collocation method, initially developed for a system of differential equations of integer order, can approximate the solutions to systems of fractional differential equations of fractional order. This method converts the system of fractional differential equations into a system of algebraic equations. These algebraic equations are then solved using the Newton-Raphson method, and the estimated values of the coefficients are then substituted in the approximation. Numerical outcomes are obtained to illustrate the simplicity and effectiveness of the proposed scheme. The numerical results demonstrate that the method is simple to use and precise. The effectiveness and consistency of the developed strategy's performance are shown in graphs and tables. The method introduces a potential technique for resolving several linear and nonlinear fractional differential equations. Mathematical software called Mathematica has been used to perform all calculations.

Keywords Fractional differential equations (FDEs), collocation technique, Riemann-Liouville fractional derivative, Fibonacci wavelet

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1. Introduction

Differential equations with arbitrary (fractional) order derivatives are called fractional differential equations (FDEs). Improved rheological models are the foundation for mathematical modelling that naturally produces FDEs. This idea generalizes the classical differential equations. Many works have been published where fractional derivatives describe better material properties, particularly in hereditary solid mechanics and viscoelasticity theory. FDEs are receiving much attention because of their frequent appearance in several applications in signal processing, biology, physics, acoustics, robotics, fluid mechanics, viscoelasticity, and engineering. Determining the exact solutions to physical phenomena is crucial to understanding

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and applying them in scientific research.

To analytically approximate the nonlinear FDEs, a few semi-analytical methods have been proposed, including the ADM [2], the HAM [3], the Homotopy asymptotic method [4], the multistep fractional differential transform method (MFDTM) [5], and the HPM [6]. Unfortunately, these techniques have limitations and cannot provide the high-precision solution of complex FDEs in many situations, especially in the unbounded domain. Therefore, there is no doubt that numerical approaches are superior to analytical ones. Since most FDEs lack precise analytical solutions, numerical techniques are frequently used. A significant area of current research has been the development of accurate and capable methods for solving FDEs. However, despite many recently formulated applied problems, the state of the art is far less advanced for generalized order equations, and only a few techniques have been proposed for numerically solving these equations. Most of these numerical schemes deal with linear single-term equations of order smaller than unity. Little efforts have been made to address nonlinear problems. Few numerical approaches have been put forward to address the system of nonlinear FDEs: the ADM method [7], the DTM method [8], the HPM method [9], the Bernoulli wavelet method (BWM) [10], and the Legendre wavelet method (LWM) [11].

Consider the SODEs is of the form:

$$\left.\begin{aligned} D^{\alpha_1}\ y_1(\xi) &= f_1(\xi, y_1, y_2, \ldots, y_n),\\ D^{\alpha_2}\ y_2(\xi) &= f_2(\xi, y_1, y_2, \ldots, y_n),\\ &\vdots\\ D^{\alpha_n}\ y_n(\xi) &= f_n(\xi, y_1, y_2, \ldots, y_n), \end{aligned}\right\} \tag{1.1}$$

where D^{α_i} is the derivative of y_i of order α_i in the sense of Riemann-Liouville fractional derivative and $0 \leq \alpha_i \leq 1$, concerning the primary constraints $y_1(0) = d_1,\ y_2(0) = d_2,\ \ldots,\ y_n(0) = d_n$.

Wavelets are among the many data types that can extract information mathematically. Sets of wavelets are necessary for the complete investigation of data. Wavelets allow reversible signal decomposition by mathematically breaking it into smaller pieces without overlaps or gaps. Sets of wavelets are valuable in wavelet-based compression/decompression techniques because retrieving the original information with little loss is desirable. Over the past 20 years, wavelet theory has been used in various signal processing applications, such as fingerprint verification, wavelet-based fingerprint storage, denoising of data, musical tone generation, etc. [12]. The orthogonal, compactly supported wavelet basis precisely approximates an increasingly higher-order polynomial. This wavelet-based representation of differential operations can be precise and stable even in areas with significant gradients or oscillations. For some of the standard mathematical problems, various wavelet collocation techniques have been used, such as the Chebyshev wavelet collocation method [13, 14], collocation method based on Bernoulli and Gegenbauer wavelets [15], Hermite wavelet collocation method [16], and Laguerre wavelet collocation method [17, 46]. Several wavelet collocation techniques are typically employed to solve fractional differential equations, which include Chelyshkov wavelets [18], Cubic B Spline [19], Genocchi wavelets [20], Taylor wavelets [21, 22], Haar wavelets [23], Bernoulli wavelets [10, 24], Hermite wavelets [25, 26], Legendre wavelets and Legendre wavelet tau method [27–29], Chebyshev wavelets [30], Gegenbaur wavelets [31] and Ultraspherical wavelets [32].