Journal of Nonlinear Modeling and Analysis
Volume 7, Number 6, December 2025, 2224–2246
https://journal.global-sci.org/jnma
DOI:10.12150/jnma.2025.2224

On Dynamics of Certain Models via Generalized Conformable Fractional Derivative

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Abstract This paper deals with the generalized conformable fractional derivative and certain interesting properties which are not compatible with Riemann-Liouville and Caputo fractional derivatives. The newly defined derivative is more efficient than other conformable fractional derivatives and the nonlocal fractional derivatives from a time perspective. To justify the claim, we provide some direct applications, such as population growth, Newton's body cooling, heat equation and susceptible-infected-removed models. Solutions obtained from models and comparison with respective previous data are demonstrated with the help of graphs or stems.

Keywords Fractional derivative, SECH-fractional derivative, fractional population growth model, fractional body cooling model, fractional heat equation, SIR model

MSC(2010) 26A33, 34A08, 35R11, 92D25.

1. Introduction

The geometry of fractional derivative of a function has been an important open problem in mathematics since 1695. In this context several definitions are proposed, where some are local and some are non-local in nature. Due to the absence of memory, ordinary derivative is considered as local derivative. In contrast, most of the popular fractional derivatives like Riemann-Liouville and Caputo possess a memory factor, so they are called non-local derivatives. Among all the non-local fractional derivatives, Riemann-liouville's definition [4, 19, 20] is the most popular and for order β it is defined as:

$$\left({}_{c}^{RL}D_{t}^{\beta}f\right)(t) = \frac{1}{\Gamma(n-\beta)}\frac{d^n}{dt^n}\int_c^t (t-x)^{n-\beta-1}f(x)dx, \qquad t > c,$$

where β is a positive real number and f is the 1^{st} order differentiable function with $n-1 \leq \beta < n$, $n \in \mathbb{N}$(the set of all positive integers). Caputo fractional derivative [4, 19, 20] of order β is defined as:

$$\left({}_{c}^{C}D_{t}^{\beta}f\right)(t) = \frac{1}{\Gamma(n-\beta)}\int_c^t (t-x)^{n-\beta-1}f^n(x)dx, \qquad t > c,$$

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where β is a positive real number and f is the n^{th} order differentiable function with $n-1 \leq \beta < n$, $n \in \mathbb{N}$.

In [14] Khalil et al. first introduced the concept of conformable fractional derivative of order β, which is local in nature and is defined as:

$$T_\beta(f)t = \lim_{\epsilon \to 0} \frac{f(t+\epsilon t^{1-\beta}) - f(t)}{\epsilon},$$

where $f : [0,\infty) \to \mathbb{R}$(the set of all real numbers), $\beta \in (0,1)$ and $t > 0$. Moving forward, the geometrical and physical properties of Khalil's conformable derivative were given in [9] and some of the major theorems and applications were given in [5–7, 11, 15, 18]. An extension of this conformable fractional derivative were given in [10] with the dynamical properties of some nonlinear partial differential equations along its solutions. Also, the dynamical properties of a non-linear fractional Schrödinger equation are compared using different known differential operators. The advantages of this derivative are that it exhibits some general properties of calculus which are not compatible with Riemann-Liouville and Caputo fractional derivative. Geometrically this idea is related to tangent vector i.e., we can approximate a tangent vector at a point where the ordinary tangent is not available. Another generalized conformable definition of β order fractional derivative was found in [3, 7], which is defined by:

$$N_F^\beta f(t) = \lim_{\epsilon \to 0} \frac{f(t+\epsilon F(t,\beta)) - f(t)}{\epsilon}, \tag{1.1}$$

where $f : [0,+\infty) \to \mathbb{R}$, F is arbitrary, $\epsilon > 0$,$t > 0$ and $\beta \in (0,1)$. For $F(t,\beta) = \exp(\beta-1)t$, we get the definition given by Katugampola [12] and denoted by $D^\beta f(t)$. Generalized conformable type derivatives are not a suitable option for a process which involves memory concept. Other than this, they are used in several well-known modeling problems and provide more precise and good outcomes.

In population modelling, there are many popular models which involve different kinds of operators. The exponential population growth model is a well-accepted model for a small time interval. The logistic model is considered when we take environmental or artificial restrictions into the consideration. In [2,6,15] exponential population model was given with Khalil's conformable fractional derivative and also the error was depicted graphically.

Another widely interpreted modelling with integer order derivative is Newton's body cooling. With the help of different non-local fractional operators, Newton's body cooling model was well explained in [5, 18], where they compared the theoretical data with the experimental data carried out under different types of liquid and fluid medium. In [7], a simulation based model was presented for Newton's body cooling, where they used Markov chain, Monte Carlo simulations(MCMC), likelihood function and different distribution techniques. The results are compared graphically.

Heat equation model is one of the most important models in mathematical physics which involves integer order partial differential equations. This concept can be extended to fractional case, which may involve different kinds of partial fractional operators. In [6, 13], heat equation was expressed in terms of Khalil's conformable derivative and its solutions are depicted.

Continuing the applications of conformable fractional derivative, the idea was utilized in SIR model, which provides the data of transmission among susceptible,