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Exact Soliton of Fifth Order (1+1) Dimensional Triple Non-Linear Partial Differential Equations on Modified Truncated Expansion Methods

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Abstract In this manuscript, modified truncated expansion method-I taking the traveling wave variable $u(x,t) = y(r) = y(kx+wt)$ and modified truncated expansion method-II introducing the traveling wave variable $u(x,t) = y(r) = y(kx - wt)$ are built up to obtain analytical solution in the form of traveling wave solutions with different frequencies and velocities that can be constructed for triple (1+1) dimensional nonlinear partial differential equations (NLPDEs) such as Sawada-Kotera equation (SKE), generalized Korteweg-de Vries equation (GKdVE) and Kaup-Kuperschmidt equation (KKE), which have been widely used in mathematical physics. The present topic minimizes the complex nature and non-integrable characteristics to obtain solutions of NLPDEs. To demonstrate the influence of the parameters, 3D plots are generated for triple NLPDEs. This content is employed in physics such as magneto sound in plasma and nonlinear optics.

Keywords Modified truncated expansion methods, triple nonlinear partial differential equations, magneto sound in plasma, nonlinear optics, traveling wave solutions

MSC(2010) 35Q51, 35Q53.

1. Introduction

NLPDEs are highly helpful in a variety of domains, including fluid dynamics, water surface gravity waves, electromagnetic radiations, and ion acoustic waves in plasma. Several techniques are adopted to search the analytical solitons to NLPDEs including methods as found in [1–4]. The propagation of waves and sound in flat surface such as capillary-gravity and shallow water and magneto sound in plasma [5] are based on modelling of GKdVE, as described in equation (1.2) in Mathematical Physics respectively. The integrable fifth-order KdVE and the necessary conditions for the nonlinearity and dispersion parameters are investigated by Hirota method [6]. Later [2] invented stationary wave solutions of binary waves for fifth-order KdVE as results of Kudryashov-expansion and sine-cosine function techniques. In order to obtain two kinds of approximate solutions a simple standard truncated expansion approach (SSTEA) has been suggested for fifth-order GKdVE [7]. The Bäcklund transformation by new GKdVE is elaborated in [8] to extract the bilinear form as

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well as structural wave phenomenon of the governing model on account of gravity field in shallow water to carry out the propagation of long waves. The exponential stability results by suitable, Lyapunov functionals on KdVE with time-dependent delay on the boundary are very much attracted in [9]. The suitable transformation is taken in [10] to reduce the GKdVE to a quadratic ordinary differential equation by implementing the new version trial equation approach for finding new solitary wave solutions. To estimate the movement of water waves the sech method is established to explore novel analytical traveling wave solutions for a nonlinear KdV in [11]. The exp-function and modified exp-function methods are implemented in [12] to explore solitary, peakon, periodic, cuspon and kink wave solutions of KdVE. Authors [13] applied the natural decomposition method for solving the time-fractional coupled KdVE. Numerical soliton KdVE in infinite dimension using the steepest descent method is found in [14]. The numerical solution of KdVE by [15] is based on the trapezoidal and implicit mid-point method via Pade technique.

The various mathematical physics concepts like quantum, optics and travelling wave solution in shallow water are mainly focused on modelling of SKE. The Hirota bilinear techniques [16,17] and modified auxiliary equation technique (MAET) and the extended direct algebraic technique (EDAT) in [18] are adopted for different analytical solitary solitons of fifth order SKE. Further, Hirota direct method [19] is investigated to exploit the -soliton solutions like, single soliton, double soliton and triple soliton solutions of the integral SKE. The independent transformation by [20] is implemented for multidimensional KdV-Sawada-Kotera-Ramani equation (KdVSKRE). The Exp-function technique has been offered to find analytical solutions of generalized SKE in [21]. The tanh-coth method is imposed on SKE to achieve explicit exact soliton [22].Yang transform (YT) with the Adomian decomposition method (ADM) and homotopy perturbation technique (HPT) are used on the seventh-order time-fractional Sawada-Kotera-Ito problem (TFSKIP) to obtain the solution in [23]. The KKE is implemented in physical sciences in different approaches like plasma physics, nonlinear optics and fluid dynamics. The analytical solution of traveling wave of KKE is found by using the Fan sub-equation method [24]. The nonlinear evolution equation particularly time fractional KKE is solved in [25] to obtain new general forms of analytical solutions by employing improved and extended $\dfrac{G'}{G}$technique that would applicable in mathematical modeling of nonlinear form. The analytical solution of KKE is extracted by implementing double $\left(\dfrac{G'}{G}, \dfrac{1}{G}\right)$ expansion approach in [26]. The authors in [27] have shown their interest towards homotopy perturbation transform method (HPTM) and Yang transform decomposition method (YTDM), to solve the fractional nonlinear seventh-order KKE through the Caputo operator. The homotopy method is addressed in [28,29] to investigate the numerical solution of KKE. Further, the one method is synthesized in [30,31] to get analytical soliton of (1+1)-Dimensional KKE. In addition to these, several computational approaches may be employed to approximate soliton solutions, including the modified auxiliary function approach [32], the natural decomposition method with Atangana-Baleanu derivative in Caputo manner and Caputo-Fabrizio [33]. There are several methods [[34–53], [61–65], [59], [60]]. Kudryashov and functional variable are implemented to obtain the analytical solutions of the strain wave equation for briefing wave propagation in micro structured solids in [54]. The soliton wave for nonlinear Schrödinger equation is also investigated in [55] to describe solitons. The $exp(-\phi(\epsilon))$ expansion and modified Kudryashov method are applied in [56] to model novel phenomena through nonlinear conformable time-fractional Zoomeron equation in (2+1)-dimension. The rational solutions are evaluated analytically to the generalized Kadomtsev-Petviashvili equation by [57]. The Hirota bilinear form and the linear superposition methods are suggested in [58] to obtain analytical solution of the (4+1)-dimensional Boiti-Leon-Manna-Pempinelli equation. As fifth order NLPDEs host the contributor towards genuine properties in several scientific and technical applications with a lot of physical characteristics [1–6], motivated by these in this manuscript we have employed two types of modified truncated expansion methods to obtain the analytical solutions of the three fifth-order NLPDEs in mathematical physics. This encourages us to study the analytical solution by implementing MATHEMATICA software. The fifth-order (1+1) dimensional SKE is represented in equation (1.1)

$$u_t + 45u^2u_x + 15u_xu_{xx} + 15uu_{xxx} + u_{xxxxx} = 0. \tag{1.1}$$

(refer [29])
Generalized fifth order Kdv equation (GKdvE) is:

$$u_t + \mu u_{xxx} + 6uu_x + \nu u_{xxxxx} = 0. \tag{1.2}$$

(refer [7])
The fifth order Kaup-Kuperschimdt equation (KKE) is :

$$-u_t + 5u^2u_x + \frac{25}{2}u_xu_{xx} + 5uu_{xxx} + u_{xxxxx} = 0. \tag{1.3}$$

(refer [24])
The plan layout of this paper is attracted in different sections. Section 2 bears theoretical illustrations of modified truncation expansion method-I. Sections 3 and 4, tackle the application of modified truncation expansion method-I for SKE and KdVE respectively. The modified truncation expansion method-II approach is suggested in Section 5 and implementation of this technique towards KKE is reflected in Section 6. The concluding remarks are appended in Section 7.

2. Modified truncation expansion method-I

Let us take modified truncation expansion method-I approach for the following NLPDE as mentioned below

$$H(u, u_t, u_x, u_{xx}, u_{xxx}, ...) = 0, \tag{2.1}$$

where H is a polynomial in the unknown function $u(x,t)$ with its nonlinear terms and higher order partial derivatives. The traveling wave variable is represented in equation (1.1) as:

$$u(x,t) = y(r) = y(kx + wt), \tag{2.2}$$

where k and w are the wave length and frequency, which permit us to reduce equation (2.1) to an ODE as given in equation (2.3) as mentioned below:

$$Q(y, y^{'}, y^{'''}, y^{v}...) = 0. \tag{2.3}$$

(a)With the placement of equation (2.4) into equation (2.3), we are able to collect the singularity of dominant terms.

$$y(r) = r^{-m}. \tag{2.4}$$

We select at least two terms with the lowest degree among all terms in equation (2.3) and the highest value of m is the pole and represented by integer M. If M is a fraction or a negative integer, we apply the following transformation: the substitution $y(r) = \phi^{\frac{\nu}{s}}(r)$ and $y(r) = \phi^M(r)$ when M is a fraction and a negative integer respectively and reselect M from the new equation.

(b)Let us take the analytical solution of equation (2.3) in the form

$$y(r) = a_0 + a_1 Q_(r) + a_2 Q^2(r) + ... + a_n Q^n(r), \tag{2.5}$$

where $r = kx + wt$ and $a_i(i = 1, 2, ..., n)$ are n constants to be calculated and

$$Q(r) = (1 + e^r)^{-1}. \tag{2.6}$$

(c) Applying MATHEMATICA, we find the first, second and higher order derivatives of $y(r)$ as follows:

$$y(r) = a_0 + a_1 Q(r) + a_2 Q^2(r) + ... + a_n Q^n(r) y'(r) = a_1 Q(r) + (a_1 + 2a_2) Q^2(r) + 2a_2 Q^3(r),$$

$$y''(r) = a_1 Q(r) + (3a_1 + 4a_2) Q^2(r) + (2a_1 + 10a_2) Q^3(r) + 6a_2 Q^4(r). \tag{2.7}$$

(d) Equation (2.1) is modified by substituting equations (2.5)-(2.7). Then, using the function $Q(r)$, we gather all terms with the same power and set the expressions equal to zero. As a consequence, simplifying the system of equations the unknown parameters are determined. This approach is easily adaptable to polynomial differential equations of any order.Based on Modified Truncation Expansion Method-I the analytical solutions of SKE and KdV equations are obtained in Section 3.

3. Application of modified truncation expansion method-I for Sawada-Kotera equation

The modified truncated expansion method-I for the two variables in general form of NLPDE can be represented by

$$H(u, u_t, u_x, u_{xx}, u_{xxx}, ...) = 0.$$

In general form, let $u(x,t) = y(kx + wt)$. It can be converted into the non-linear ordinary differential equation (ODE):

$$wy^{'} + 45ky^2y' + 15k^3 y^{'} y^{''} + k^5 y^{\nu} = 0. \tag{3.1}$$

Let us take

$$y(z) = a_0 + a_1 q + a_2 q^2. \tag{3.2}$$

Substituting equation (3.2) into equation (3.1) and comparing the coefficients of Q^i, where $i = 1, 2, .., .7$, the following algebraic equations (3.3)-(3.9) are obtained:

$$-wa_1 - 45ka_0^2 a_1 - 15k^3 a_0 a_1 - k^5 a_1 = 0. \tag{3.3}$$

$$w(a_1-2a_2)+45ka_0^2(a_1-2a_2)-90ka_0a_1^2-30k^3a_1^2+15k^3a_0(7a_1-8a_2)$$
$$+k_5(31a_1--32a_2)=0. \tag{3.4}$$

$$2wa_2+90ka_0^2a_2+90ka_0a_1(a_1-2a_2)-45ka_1(a_1^2+2a_0a_2)-15k^3a_1(4a_2-3a_1)$$
$$+15k^3a_1(a_1-2a_2)+15k^3a_0(38a_2-12a_1)+15k^3a_1(7a_1-8a_2)-15k^3a_1a_2$$
$$+k^5(422a_2-180a_1)=0. \tag{3.5}$$

$$180ka_0a_1a_2+45k(a_1^2+2a_0a_2)(a_1-2a_2)-90ka_1^2a_2-15k^3a_1(2(a_1-10a_2)$$
$$+15k^3(a_1-2a_2)(4a_2-3a_1)+30k^3a_2a_1+15k^3a_0(6a_1-54a_2)$$
$$+15k^3a_1(38a_2-12a_1)+15k^3a_2(7a_1-8a_2)+k^5(390a_1-1710a_2)=0. \tag{3.6}$$

$$90ka_2(a_1^2+2a_0a_2)+90ka_1a_2(a_1-2a_2)-45ka_1a_2^2-90k^3a_1a_2$$
$$+15k^3(a_1-2a_2)(2a_1-10a_2)+30k^3a_2(4a_2-3a_1)+360k^3a_0a_2$$
$$+15k^3a_1(6a_1-54a_2)+15k^3a_2(38a_2-12a_1)+k^5(3000a_2-360a_1)=0. \tag{3.7}$$

$$180ka_1a_2^2+45ka_2^2(a-1-2a_2)+90k^3a_2(a_1-2a_2)+30k^3a_2(2a_1-10a_2)$$
$$+360k^3a_1a_2+15k^3a_2(6a_1-54a_2)+k^5(120a_1-2400a_2)=0. \tag{3.8}$$

$$90ka_2^3+540k^3a_2^2+720k^5a_2=0. \tag{3.9}$$

With the help of MATHEMATICA, the set of equations (3.3) -(3.9) are solved and the following different cases arise.

Case 1:
$a_0=\frac{k^2}{3}, a_1=4k^2, a_2=-4k^2, w=-k^5$, where $r=(kx+wt)=k(x-k^4t)$.
$y(r)=\frac{k^2}{3}(1+12q-12q^2)$ where $q=\frac{1}{(1+e^r)}$.
Taking $k=1.5$, the 3D graph is shown in Fig-1.

Case 2:
$a_0=k^2, a_1=2k^2, a_2=-2k^2, w=-61^5$, where $r=(kx+wt)=k(x-k^4t)$.
$y(r)=k^2(1+2q-2q^2)$, where $q=\frac{1}{(1+e^r)}$.
Taking $k=0.5$, the 3D graph is shown in Fig-2.

Case 3:
Considering $a_1=2k^2, a_2=-2k^2, a_0=1, w=-(k^5+15k^3a_0+45ka_0^2)$, where $r=(kx+wt)=kx-(k^5+15k^3a_0+45ka_0^2)t$;
and $y(r)=1+2k_2(q-q^2)$, where $q=\frac{1}{(1+e^r)}$.
Taking $k=0.5$, the 3D graph is shown in Fig-3.

4. Application of modified truncation expansion method-I on fifth-order Kdv equation

In general form, let $u(x,t)=y(kx+wt)$, where $w\neq 0$ and equation (1.2) takes the nonlinear differential equation as given in equation (4.1) as below:

$$wy^{'}+\mu k^3y^{'''}+6kyy^{'}+\nu k^5y^{\nu}=0. \tag{4.1}$$

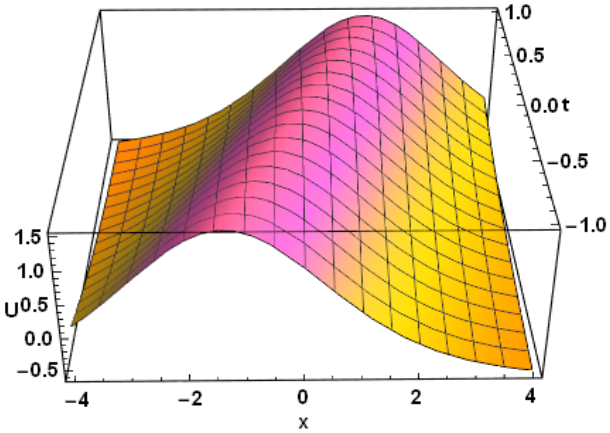

Figure 1. Analytical solution representation of Sawada-Kotera for Case 1 with $-4 \leq x \leq 4$ and $-1 \leq x \leq 1$.

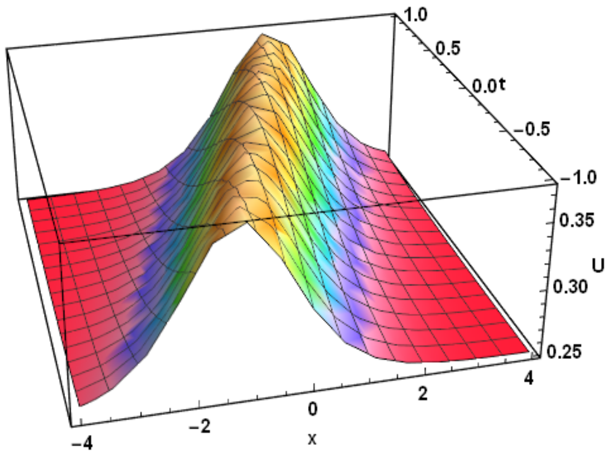

Figure 2. Analytical solution representation of Sawada-Kotera for Case 2 with $-4 \leq x \leq 4$ and $-1 \leq x \leq 1$.

Let us take

$$y(r) = a_0 + a_1 Q + a_2 Q^2. \tag{4.2}$$

Substituting equation (4.2) into equation (4.1) and comparing the coefficients of Q^i, where $i = 1, 2, \dots, 7$, the following algebraic equations are obtained.

$Q : -wa_1 - \mu k^3 a1 - 6ka_0a_1 - \nu k^5 a_1 = 0.$

$Q^2 : w(a_1 - 2a_2) + \mu k^3(7a_1 - 8a_2) + 6ka_0(a_1 - 2a_2) - 6ka_1^2 + \nu k^5(31a_1 - 32a_2) = 0.$

$Q^3 : 2wa_2 + \mu k^3(38a_2 - 12a_1) + 12ka_0a_2 + 6ka_1(a_1 - 2a_2) - 6ka_1a_2$
$\quad + \nu k^5(422a_2 - 180a_1) = 0.$

$Q^4 : \mu k^3(6a_1 - 54a_2) + 12ka_1a_2 + 6ka_2(a_1 - 2a_2) + \nu k^5(390a_1 - 1710a_2).$

$Q^5 : \nu k^5(3000a_2 - 360a_1) = 0.$

$Q^6 : \nu k^5(120a_1 - 2400a_2) = 0.$

$Q^7 : 720\nu k^5 a_2 = 0.$

Case 1: $a_2 \longrightarrow -a_1, w \longrightarrow \dfrac{1}{2}k(12a_0 + a_1),$

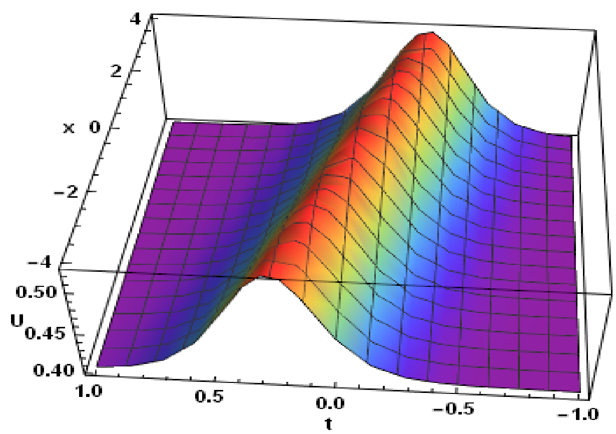

Figure 3. Analytical solution representation of Sawada-Kotera for Case 3 with $-4 \le x \le 4$ and $-1 \le x \le 1$.

$\mu \longrightarrow \dfrac{a_1}{2k_2}, \nu \longrightarrow 0, a_0 = 0.2, a_1 = 0.3,$
$r = kx - wt = kx - \dfrac{kt}{2}(12a_0 + a_1),$
$y(r) = a_0 + a_1(q - q^2)$, where $q = \dfrac{1}{1+e^r}$.
Taking $k = 0.8$, the 3D graph is shown in Fig-4.

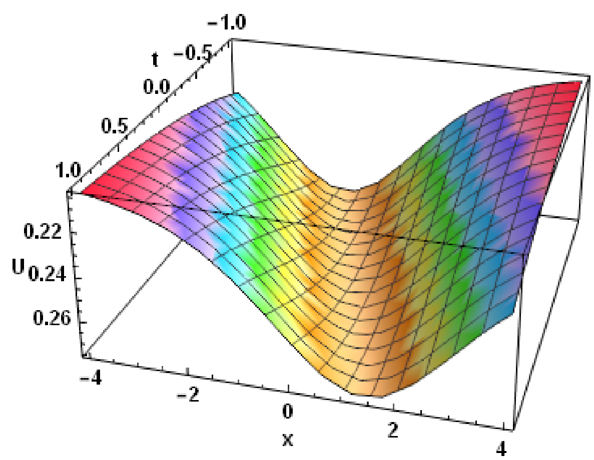

Figure 4. Analytical solution representation of fifth-order Kdv equation with $-4 \le x \le 4$ and $-1 \le x \le 1$.

5. Modified truncation expansion method-II

Let us propose the modified truncation expansion method-II approach for the following NLPDE as given below:

$$H(u, u_t, u_x, u_{xx}, u_{xxx} \cdots) = 0. \tag{5.1}$$

Here, $u(x,t)$ with its nonlinear terms and higher order derivatives is embedded in H. The traveling wave variable is represented in equation (1.3) as:

$$u(x,t) = y(r) = y(kx - wt), \tag{5.2}$$

where k and w are the wave length and frequency, allowing us to reduce equation (5.1) to an ODE as given in equation (5.3)

$$Q(y, y^{'}, y^{'''}, y^{(\nu)} \cdots) = 0. \tag{5.3}$$

(a) With the placement of equation (5.4) in equation (5.3), we are able to collect the singularity of dominant terms

$$y(r) = r^{-m}. \tag{5.4}$$

We select at least two terms with the lowest degree among all terms in equation (5.3) and the highest value of m is the pole and represented by the integer M. If M is a fraction or a negative integer, we apply the following transformation:: the substitution $y(r) = \phi^{\frac{\nu}{s}}(r)$ and $y(r) = \phi^{M}(r)$ when M is a fraction and a negative integer respectively and reselect M from the new equation.

(b) Let us take the analytical solution of equation (5.3) in the form

$$y(r) = a_0 + a_1 Q(r) + a_2 Q^2(r) + ... + a_n Q^n(r). \tag{5.5}$$

where $r = kx - wt$ and $a_i (i = 1, 2 \ldots, n)$ are n constants to be calculated and

$$Q(r) = (1 + e^r)^{-1}. \tag{5.6}$$

(c) Applying MATHEMATICA, we find the first, second and higher order derivatives of $y(r)$ as follows:

$$\begin{aligned} y(r) =& a_0 + a_1 Q(r) + a_2 Q^2(r) + ... + a_n Q^n(r), \\ y^{'}(r) =& - a_1 Q(r) + (a_1 - 2a_2) Q^2(r) + 2a_2 Q^3(r), \\ y^{''} =& a_1 Q(r) + (4a_2 - 3a_1) Q^2(r) + (2a_1 - 10a_2) Q^3(r) + 6a_2 Q^4(r). \end{aligned} \tag{5.7}$$

(d) Equation (4.1) is modified by substituting equations (5.5)-(5.7). Then, using the function Q(r), we gather all terms with the same power and set the expressions equal to zero. As a consequence, the evaluation of unknown parameters can be investigated by simplifying the system of equations. Solving this system yields the values of the unknown parameters. This approach is easily adaptable to polynomial differential equations of any order. Based on the modified truncation expansion method-II, the analytical solutions of KKE equations are obtained in Section 5.

6. Application of modified truncation expansion method-II on Kaup-Kupershimdt equation

In this section, the procedure for the modified truncation expansion method-II in Section 5 is implemented for KKE which is represented in equation (1.3). Applying the procedure (a)-(d) of algorithm of Section 5 and using travelling wave variable

$u(x,t) = y(r)$ where $r = kx - wt$ in equation (1.3), it can be converted into nonlinear ordinary differential equation (NLODE) of the form

$$wy^{'}(z) + 5ky^2(z)y^{'}(z) + \frac{25}{2}k^3y^{'}(z)y^{''}(z) + 5k^3. \tag{6.1}$$

After integrating by parts and applying the substitution method into equation (5.1), it takes the form

$$wy(z) + \frac{5}{3}ky^3(z) + \frac{15}{4}k^3y^{'}(z)^2 + 5k^3y(z)y^{''}(z) + k^5y^{iv}(z) + c_1 = 0. \tag{6.2}$$

Let us take

$$y(r) = a_0 + a_1Q + a_2Q^2. \tag{6.3}$$

Substituting equation (6.3) into equation (6.2) and comparing the coefficients of Q^i, where $i = 1, 2, ..., 7$, the following algebraic equations are obtained.

$a_0w + \frac{5}{3}ka_0^3 + c_1 = 0.$

$a_1w + 5ka_0^2a_1 + 5k^3a_0a_1 + k^5a_1 = 0.$

$a_2w + 5k(a_0a_1^2 + a_0^2a_2) + \frac{15}{4}k^3a_1^2 + 5k^3(4a_0a_2 - 3a_0a_1 + a_1^2) + k^5(16a_2 - 15a_1) = 0.$

$\frac{5}{3}K(a_1^3 + 6a_0a_1a_2) - \frac{15}{4}k^3(2a_1^2 - 4a_1a_2) + 5k^3(2a_0a_1 - 10a_0a_2 + 5a_1a_2 - 3a_1^2)$
$+ k^5(50a_1 - 130a_2) = 0.$

$5k(a_1^2a_2 + a0a_2^2) + \frac{15}{4}k^3((a_1 - a_2)^2) - 4a_1a_2) + 5k^3(6a_0a_2 + 2a_1^2 - 10a_1a_2 + 4a_2^2$
$- 3a_1a_2) + k^5(330a_2 - 60a_1) = 0.$

$5ka_1a_2^2 + 15k^3(a_1a_2 - 2a_2^2) + 5k^3(8a_1a_2 - 10_2^2) + k^5(24a_1 - 336a_2) = 0.$

$\frac{5}{3}ka_2^3 + 15k^3a_2^2 + 30k^3a_2^2 + 120k^5a_2 = 0.$

Case 1: $a_0 = -\frac{k^2}{4}, a_1 = 3k^2, a_2 = -3k^2, c_1 = \frac{k^7}{96}, w = -\frac{k^5}{16},$
$r = kx - wt = kx + \frac{k^5}{16}t,$
$y(z) = -\frac{k^2}{4} + 3k^2(Q - Q^2)$, where $Q = (1 + e^(r))^{-1}$.
Taking k=0.8, the analytical solution of 3D is shown in Fig-5.

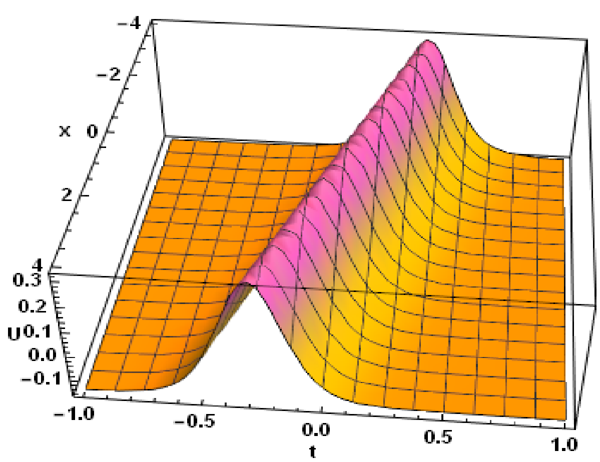

Figure 5. Analytical solution of Kaup-Kupershmidt equation for Case 1 with $-4 \leq x \leq 4$ and $-1 \leq x \leq 1$.

Case 2: $a_0 = -2k^2,\ a_1 = 24k^2,\ a_2 = -24k^2,$
$c_1 = -\dfrac{26}{3}k^7, w = -11k^5,\ z = kx + 11k^5 t,\ y(r) = -2k^2 + 24k^2(q - q^2),$
where $q = \dfrac{1}{1+e^r}$. Taking k=1, the analytical solution of 3D graph is shown in Fig-6.

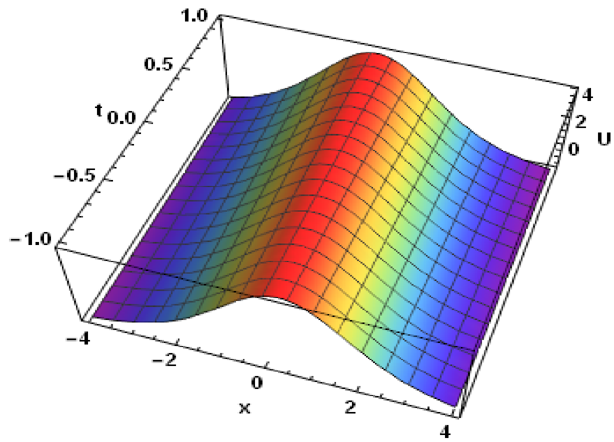

Figure 6. Analytical solution representation of Kaup-Kupershmidt equation for Case 2 with $-4 \leq x \leq 4$ and $-1 \leq x \leq 1$.

7. Conclusions

In this manuscript, the SKE, KdvE and KKE have been analyzed to obtain new analytical traveling wave solutions with different frequencies and velocities. Then modified truncation expansion method-I and modified truncation expansion method-II techniques have been employed to obtain several types of solutions to the problems under consideration. The analytical solutions of solitons have been surveyed for different justified parameters to strengthen the quality of the present work. In future work, the analytical methods presented in this manuscript can be implemented to other types of NLPDEs for computing the solitons of a suitable model that would be applicable in physical sciences.

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