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Fractional Order Solution to Analyze Tumor-Immune Cells by Using Hybrid Natural Transform Method and Residual Power Series Method*

Ashrita Patra[1] and Tapasi Pasayat[1,†]

Abstract The purpose of this study is to examine the impact of fractional-order chemotherapy drug diffusion on tumor-immune cell growth. To address the proposed system of fractional-order tumor-immune cell dynamics, two distinct and robust methods have been employed. Initially, the Hybrid Natural Decomposition method, an analytical approach that merges two effective techniques – Natural Transform and the Adomian Decomposition Method has been applied. Subsequently, the Residual Power Series method, a numerical technique, has been employed to derive the solution in series form. The graphical comparison of the results obtained from both methods with those from classical-order solutions has been provided. The findings clearly demonstrate that the proposed approaches offer a reliable and accurate means of understanding the intricate interactions between tumor growth and immune cell activity.

Keywords Chemotherapy, cancer, Residual Power Series method, Natural Transform method, tumor immune cell, fractional derivative, Adomian polynomials

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1. Introduction

The current challenge of cancer, in terms of sickness and mortality, remains one of the most crucial matters in modern medicine. Despite notable advancements in both fields, the intricacy of tumor development and the complex interplay between cancer cells and the immune system continue to manifest significant obstacles in cancer research and treatment [19, 21]. The emerging field of tumor-immune surveillance has recently drawn surging attention due to its unique perspective on the body's natural defenses against cancer growth [2]. The exact cause of most tumors remaining unknown, specific risk factors like exposure to radiation, tobacco usage, and genetic mutations have been linked to their development [23]. The interaction between tumor cells and immune cells, such as cytotoxic T-cells (CD8+ T cells),

†the corresponding author.
Email address: ashrita.patra@gmail.com, tapashya.94@gmail.com
[1]School of Mathematics, Gangadhar Meher University, Amruta Vihar, Sambalpur, India.
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natural killer (NK) cells, and drug cells, is a multifaceted and dynamic process that plays a critical role in the progression of cancer and the response to treatment. CD8+ T lymphocytes, also referred to as cytotoxic T cells, play a crucial role in the adaptive immune system. They have the ability to identify and remove tumor cells that exhibit particular antigens on their surfaces. This recognition process takes place through antigen presentation, during which tumor cells present tumor-specific antigens on their surface in association with major histocompatibility complex class I (MHC-I) molecules.

Based on prior studies [7, 22], equation (1.1) depicts the fractional order relationship between tumor cells and the anti-tumor immune system, as well as the impact of chemotherapy on both the tumor cells and the immune system in caputo sense.

$$\begin{aligned}
{}^{C}D_t^{\varkappa} N(t) &= \left(\phi N(1-\lambda N) - \mu_1 NT - \delta_1 UN\right), \\
{}^{C}D_t^{\varkappa} L(t) &= \left(\alpha NT - \beta L - \eta_1 LT - \delta_2 UL\right), \\
{}^{C}D_t^{\varkappa} T(t) &= \left(\sigma T(1-\tau T) - \mu_2 NT - \eta_2 LT - \delta_3 UT\right), \\
{}^{C}D_t^{\varkappa} U(t) &= \left(\xi - rU\right).
\end{aligned} \tag{1.1}$$

The research affirms that the model explains how tumor cell and immune system population change under chemotherapy. T represents the population of tumor cells at time t, while N and L respectively represent the population of Natural-Killer cells and cytotoxic T-cells, and U represents the amount of drug at the tumor site. According to the model, the decrease in tumor population is caused by both immune effector cells and chemotherapy. This decrease is a result of the degradation process, which involves the consumption that occurs when tumor cells are killed due to chemotherapy, leading to a subsequent decrease in the population of effector cells. In addition, because chemotherapy drugs impact both tumor cells and immune effector cells using a mass-action mechanism, administering a higher constant dosage of the drug can lead to increased depletion of both the tumor and immune effector cells. The description and values of the parameters used in Eq.(1.1) as per the existing literature [7, 22] are given in Table 1.

By combining the two powerful techniques, Natural Transform(NT) and Adomian Decomposition Method(ADM), the technique Hybrid Natural Decomposition Method(HNDM), offers a unique solution to the complex dynamics of tumor-immune cell interactions, which has not been extensively explored in the existing literature. The HNDM integrates the strengths of NT and ADM, providing an analytical framework that is both efficient and precise for tackling fractional-order differential equations. This allows for a deeper understanding of tumor growth and immune response at fractional orders, a dimension that conventional integer-order models fail to capture.

In addition, the application of the Residual Power Series(RPS) method offers an effective numerical technique to obtain solutions in series form. The use of these methods introduces a new paradigm for solving fractional-order models, enabling a more accurate representation of the dynamic and nonlinear processes governing tumor-immune interactions. These novel approaches not only enhances the precision of predictions in cancer research but also provides valuable insights into the complex relationships among chemotherapy drug diffusion, tumor growth and immune cell responses, opening new avenues for improving cancer treatment strategies.