A Semi-Analytical Study of a Non-Linear Initial Value Problem for the Lassa Fever Model

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Abstract Mathematical modelling plays a crucial role in comprehending and exercising control over Lassa fever outbreaks. The Homotopy Analysis Method (HAM) is employed to address the system of nonlinear differential equations and generate a semi-analytical solution for the Lassa disease Susceptible-Exposed-Infected-Recovered (SEIR) model. Comparing with numerical simulations, the HAM technique yielded extremely precise approximate solutions, proving its accuracy and efficiency. The numerical simulation is carried out using MATLAB programming. There is a good agreement between the numerical simulation and the approximate analytical results. We also examine the effects of changing model parameters on the different compartments, obtaining insightful knowledge on how the model behaves in various scenarios. This analysis is crucial for understanding how variations in transmission rates, recovery rates, and other key factors affect the overall dynamics of Lassa fever, thereby guiding more effective public health strategies and disease management. Moreover, it underscores the potential of the Homotopy Analysis Method (HAM) as a powerful tool for exploring epidemic models and formulating control strategies.

Keywords Epidemic model, Lassa fever, non-linear initial value problem, homotopy analysis method (HAM), numerical simulation

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1. Introduction

The Lassa virus causes Lassa fever, an acute viral disease. Humans are frequently infected with Lassa virus from other animals, particularly the Natal multimammate mouse. In West Africa, which includes Guinea, Ghana, Sierra Leone, Nigeria, and Liberia, Lassa fever is comparatively prevalent [12]. Lassa fever takes two to twenty-one days to incubate. Eighty percent of infected people have minimal or no symptoms. Many virus-infected people do not show any symptoms. Headaches,

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fever, weakness, vomiting, and muscle pains are some examples of these minor symptoms [7]. 20% of people may get more serious symptoms like vomiting, chest pain, breathing issues, and bleeding gums. Hearing loss is one of the long-term consequences. Approximately 1% of Lassa virus infections are fatal.

Humans contract the Lassa virus mostly through direct contact with infected rats or by consuming food or household goods tainted with the urine or faeces of an infected Mastomys rat. Direct contact between individuals can then result in spread. As of 2023, there is no human vaccination [24]. Isolating sick individuals and reducing interaction with the rats are necessary for prevention. Food and grain should be kept in sealed containers, good personal hygiene should be promoted, and trash should be disposed of far from the house to maintain clean homes as further measures to prevent the spread of illness. Goggles, lab coats, gloves, and masks are recommended while interacting with an infected individual in order to prevent coming into contact with bodily fluids like blood [38]. Improving symptoms and treating dehydration are the goals of treatment.

Abidemi and Owolabi [2] used the least squares approach to fit the model to the Nigeria Centre for Disease Control database, which contained the matching cumulative number of weekly reported cases. The next-generation matrix approach is used to determine the model's fundamental reproduction number, and sensitivity analysis is performed to further identify the most sensitive parameters influencing the dynamics of disease transmission. In order to account for five time-dependent control variables, Abidemi et al. [3] extended the seven-dimensional deterministic model of LF dynamics. Using Pontryagin's maximal principle and optimal control theory, Abidemi et al. [3] were able to determine the prerequisites for the existence of an optimal control quintuple. Additionally, he conducted cost-effectiveness and efficiency analyses to identify the least expensive control strategy that can be used to stop the spread of LF in the population and to identify the most effective interventions among the group of various control strategies under consideration. Abiola et al. [4] calculated the fundamental reproduction number using the next-generation matrix approach. Additionally, Pontryagin's Maximum Principle was used to derive the associated optimality system, and the Lassa fever pandemic scenario was articulated as an optimal control issue.

In order to determine the most sensitive epidemiological parameters to lessen or mitigate the Lassa virus spread in Nigeria and beyond, Abdulhamid et al. [1] employed qualitative analysis to demonstrate the existence of backward bifurcation and the partial rank correlation coefficient. For Lassa fever, Agbi et al. [5] developed and investigated a non-linear mathematical compartmental model that incorporated human and rodent populations together with an infected environment. To determine the reproduction number, the next-generation method was applied and also he depicted rodent predation utilizing the Holling type II functional response.

Ajala et al. [6] tested and validated the solution's uniqueness using an invariant condition, and they calculated the reproduction number using the next generation matrix. Ayoade et al. [8] used numerous mathematical theorems to investigate and prove the model's validity. A threshold for the eradication of the disease and its equilibrium were determined. In order to determine the required and sufficient criteria for the model's equilibria to be stable both locally and globally, the stability was examined. Additionally, sensitivity analysis was performed to examine the relative contributions of different parameters to the treatment and spread of Lassa disease.