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Bifurcation Analysis of Predator-Prey Mathematical Model with Resource-Limited Growth and Predator Switching Behavior

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Abstract Ecology examines the interactions between organisms and their environment, with a particular focus on population dynamics, resource availability, and predator-prey relationships. This study presents a mathematical model designed to investigate the interactions between two prey populations, one in an unprotected region and the other in a protected region, along with a predator population and shared resource availability. The model employs nonlinear differential equations to capture processes such as prey growth, predation, and resource utilization. By identifying equilibrium points and conducting eigenvalue analysis, we assess the system's stability. Numerical simulations demonstrate a range of outcomes, including stable states, cyclic behavior, and population collapse, depending on ecological conditions (parametric values). Biologically, predator-prey coexistence equilibria may lose stability, shifting to extinction or dominance scenarios. This makes bifurcation point. Bifurcation analysis reveals how competition and predation impact stability, with critical points marking transitions between coexistence, oscillations, or extinction. These results underscore the intricate balance of ecological forces and emphasize the significance of resource management and conservation in preserving ecosystem stability.

Keywords Ecology, predator-prey model, resource availability, stability analysis, local bifurcation

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1. Introduction

Ecology, a discipline within biology, examines the distribution of organisms and their interactions with the environment and one another [1–3]. The evolution and growth of species are shaped by a range of factors, including population density, resource availability, interspecies interactions, and the ecological and environmental conditions of their habitats [4]. The incorporation of mathematical frameworks into

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ecological research has given rise to the field of mathematical ecology, which utilizes mathematical models and equations to describe and predict the temporal dynamics of interacting species. The study of predator-prey interactions has been a fundamental area of ecological research, with mathematical modeling playing a crucial role in understanding population dynamics and ecosystem stability [1,5]. Classical models, such as the Lotka-Volterra equations, provide foundational insights but are often limited in their ability to capture the complexity of real-world ecosystems [4, 6, 7]. Natural habitats are heterogeneous, with prey distributed across areas of varying predation risk and resource availability, such as reserved (low risk, resource-rich) and unreserved (high risk, resource-limited) zones [8]. Resources play a crucial role in shaping prey distribution, reproduction, and survival, further influencing predator-prey dynamics. Classical models like Lotka-Volterra assume homogeneous environments and linear dynamics, oversimplifying real ecosystems [9, 10]. As reported in [11–13] the Holling Type-II functional response improved upon this by incorporating saturating predation rates but overlooked spatial heterogeneity, resource limitation, and predator switching where predators shift to abundant prey to optimize energy intake. Spatially explicit models better capture ecological complexity, emphasizing the role of reserved zones and resource distribution in stabilizing prey populations and informing conservation strategies. Additionally, resource availability, a critical factor influencing prey reproduction and predator survival, is inherently dynamic and subject to environmental fluctuations [14, 15]. To better understand these interactions, models must incorporate spatial structure, resource dynamics, and predator-prey interactions. This study extends traditional models by introducing a framework where prey are divided into unreserved and reserved regions, predators roam freely, and resource availability fluctuates dynamically, offering a more realistic representation of ecological systems. The main challenge is understanding how spatially structured habitats and dynamic resources influence predator-prey interactions and population stability. Traditional models, assuming homogeneous environments and constant resources, fail to capture the complexity of natural systems, limiting their predictive power. This study addresses this gap by developing a mathematical model that integrates spatial heterogeneity, predator-prey dynamics, and resource variability to explore these interactions in structured ecological contexts.

Existing predator-prey models are constrained by simplifying assumptions that overlook critical aspects of real world ecosystems [16, 17]. These include spatial heterogeneity, where reserved zones lower predation risk and unreserved zones face higher predation [18], as well as temporal resource dynamics affecting prey growth and predator survival. Traditional models also overlook non-linear interactions between prey across regions and predators moving through the landscape. To address these gaps, this study introduces a comprehensive mathematical model incorporating spatially structured prey, dynamic resources, and predator-prey interactions. This framework aims to enhance understanding of ecological dynamics, improve population predictions, and support conservation strategies in structured, dynamic habitats.

2. Mathematical model assumption

This document presents a prey-predator model that integrates logistic resource dynamics and a protected prey population. The model captures the interactions be-