Journal of Nonlinear Modeling and Analysis
Volume 7, Number 6, December 2025, 2379–2391
https://journal.global-sci.org/jnma
DOI:10.12150/jnma.2025.2379

Application of the Functional Variable Method to Some Nonlinear Evolution Equations

Patanjali Sharma[1,†]

Abstract The functional variable method is a highly effective approach for deriving exact solutions to nonlinear evolution equations. It offers broad applicability in addressing nonlinear wave equations. In this paper, the functional variable method is employed to obtain soliton solutions for the Radhakrishnan-Kundu-Lakshmanan (RKL) equation and the Landau-Ginzburg-Higgs equation. Exact solutions play a crucial role in uncovering the internal mechanisms of physical phenomena. Graphical representations of the obtained optical soliton solutions are provided to illustrate some of their physical parameters.

Keywords Solitary wave solution, Radhakrishnan-Kundu-Lakshmanan equation, Landau-Ginzburg-Higgs equation, functional variable method

MSC(2010) 35Q51, 35C08, 35J10.

1. Introduction

The solitary wave solutions to nonlinear evolution equations (NLEEs) are highly significant due to their potential applications in various physical fields, including neural physics, chaos theory, diffusion processes, and reaction dynamics, among others. The solitary wave solutions to nonlinear evolution equations (NLEEs) in mathematical physics have seen significant advancements over the past decades. The direct pursuit of exact solutions for PDEs has gained increasing interest, partly due to the availability of computer algebra systems like Maple and Mathematica, which facilitate complex and labor-intensive algebraic computations. There are several schemes, for instance, Haar wavelet method [1], homotopy asymptotic method [2], extended tanh technique [3], variational iteration method [4] the modified Kudryashov method [5], the auxiliary equation method [6, 7], the (G'/G)-expansion method [8–10], the first integral method [11, 12], the new generalized (G'/G) expansion method [13], the improved Kudryashov method [14], the tanh-method [15, 16], the extended trial equation method [17], the (G'/G, 1/G)-expansion method [18], the sine-Gordon equation expansion method [19], the improved F-expansion method [20], the modified simple equation method [21, 22], the improved Bernoulli sub-equation function method [23], the Darboux transformation method [24–26], the Modified Sub-Equation method [27], the Sardar sub equation method [28], etc. which have been used for searching stable closed-form wave solutions to NLEEs.

†the corresponding author.
Email address:sharmapatanjali@rediffmail.com(Patanjali Sharma)
[1]Department of Education in Science and Mathematics, Regional Institute of Education, NCERT, Ajmer 305004, India.

The Radhakrishnan-Kundu-Lakshmanan (RKL) equation is a generalized form of the nonlinear Schrödinger equation that describes the dynamics of soliton propagation in polarization-preserving fibers. This equation was first introduced in 1999 [29]. Over the years, it has been extensively studied by various researchers. For instance: Arshed et al. [30] derived optical solitons for the RKL equation with full nonlinearity. Biswas et al. [31] analyzed the RKL equation using the extended trial function scheme. Gaxiola and Biswas [32] explored the RKL equation via the Laplace-Adomian decomposition method. Ghanbari et al. [33] obtained exact optical solitons of the RKL equation under Kerr law nonlinearity. Ganji et al. [34] examined the nonlinear RKL equation in detail. Elsherbeny et al. [35] applied the improved modified extended tanh-function method to study the RKL equation. These studies highlight the diverse mathematical approaches and the significance of the RKL equation in understanding soliton dynamics.

The Landau-Ginzburg-Higgs equation, developed by Lev Davidovich Landau and Vitaly Lazarevich Ginzburg, has broad applications in explaining phenomena such as superconductivity and drift cyclotron waves in radially inhomogeneous plasma, as well as coherent ion-cyclotron waves. Various methods have been employed to derive unique soliton solutions for the integrable nonlinear evolution equation (NLEE). For example, Bekir and Unsal [36] applied the first integral method to analyze Landau-Ginzburg-Higgs equation and obtained exponential function solutions. Iftikhar et al. [37] explored different types of analytic solutions using the (G'/G,1/G)-expansion method, yielding general soliton solutions and kink-shaped solitons under varying parametric conditions. Islam and Akbar [38] utilized the IBSEF method to derive several stable solution types. Ahmad K et al. [39] used power index method to derive exact solutions of Landau-Ginzburg-Higgs equation. These studies demonstrate the versatility of mathematical approaches used to analyze the Landau-Ginzburg-Higgs equation and its relevance in physical applications.

The powerful and effective method for finding exact solutions of nonlinear evolution equations was proposed in [40, 41] by Zerarka et al., which is called the functional variable method. Recently, Babajanov [42–44] applied this method to obtain soliton solutions of various NLEEs.

In this paper, we extend the application of functional variable method to solve the Radhakrishnan-Kundu-Lakshmanan (RKL) and Landau-Ginzburg-Higgs equation. In Section 2, we propose the basic idea of the method for finding exact travelling wave solutions of NLEEs. In Section 3, we establish the exact travelling wave solution for Radhakrishnan-Kundu-Lakshmanan (RKL) and Landau-Ginzburg-Higgs equation. Finally, in Sections 4 and 5, graphical representation of the equations and conclusions are given.

2. Basic idea of the Functional Variable Method

Consider the nonlinear partial differential equation(NLPDE) of the form:

$$P(u, u_t, u_x, u_y, u_{tt}, u_{xx}, u_{xy}, u_{xt}, u_{yt}, u_{yy}...) = 0, \tag{2.1}$$

where P is a polynomial in $u = u(x, y, t)$ and its partial derivatives. The main steps of this method can be described as follows:

Step 1. In order to find the travelling wave solution of Eq. (2.1), we introduce the