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Existence of Multiple Weak Solutions for Nonlinear Problems on the Sierpiński Gasket

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Abstract. In this paper, we study the existence of at least two, three and infinitely many solutions for nonlinear problems on the Sierpiński gasket, modelling some physical phenomena such as reaction-diffusion problems, elastic properties of fractal media and flow through fractal non-smooth domains. We will obtain the existence of two weak solutions when nonlinear term $f(x,t)$ is non-negative, when it is non-positive in neighbored of zero and otherwise is positive we will show the existence of three weak solutions, and when it is odd we will get the existence of infinitely many solutions. The results are proved by using some critical point theorems.

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1 Introduction

In this paper, we will study the existence of multiple weak solutions for the following problem

$$\begin{cases} \Delta u(x)+\alpha(x)u(x)=\lambda f(x,u(x)), & x\in V\setminus V_0, \\ u=0, & x\in V_0, \end{cases} \tag{1.1}$$

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where V stands for the Sierpiáski gasket in $(\mathbb{R}^{N-1},|.|)$, $N\geq 2, V_0$ is its intrinsic boundary (consisting of its N corners), $\lambda>0$ a real number, Δ denotes the weak Laplacian on V and $\alpha\in L^1(V)$ and $f:V\times\mathbb{R}\longrightarrow\mathbb{R}$ is a continues function.

The elliptic equation (1.1) models some physical phenomena such as reaction-diffusion problems, elastic properties of fractal media and flow through fractal non-smooth domains. In the last year a great attention has been focused by many authors on the study of elliptic equations on fractal and in particular on the Sierpiński gasket. For instance [1–10] on methods variational in the space Sierpiáski gasket. In [7], Hu analyzed the existence of p-pairs of non-trivial solutions of the problem (1.1) in the case where V is the Sierpiáski gasket in $\mathbb{R}^2$. Bonanno et al., in [2], using variational methods have discussed the existence of a sequence of solutions for an eigenvalue Dirichlet problem on the Sierpiński gasket, which it is the following Dirichlet problem

$$\begin{cases}\Delta u(x)+\alpha(x)u(x)=\lambda g(x)f(u(x)), & x\in V\setminus V_0,\\ u=0, & x\in V_0,\end{cases} \tag{1.2}$$

where V stands for the Sierpiński gasket, V_0 is its intrinsic, Δ denotes the weak Laplacian on V and λ is a positive real parameter. They have assumed that $f:\mathbb{R}\longrightarrow\mathbb{R}$ be a continuous function and the variable potential $\alpha,g:V\longrightarrow\mathbb{R}$ satisfy the following conditions:

(h_1) $\alpha\in L^1(V,\eta)$ and $\alpha\leq 0$ almost everywhere in V;

(h_2) $g\in C(V)$ whit $g\leq 0$ and such that the restriction of g to every open subset of V is not identically zero. In [9] Molica Bisci et al. applying variational methods and critical point theory have investigated the existence of one non-zero strong solution for the following elliptic equations defined on the Sierpiński gasket

$$\begin{cases}\Delta u(x)=\lambda\alpha(x)f(u(x)), & x\in V\setminus V_0,\\ u=0, & x\in V_0,\end{cases}$$

where V stands for the Sierpiński gasket in $(\mathbb{R}^{(N-1)},|.|), N\geq 2, V_0$ is its intrinsic boundary (consisting of its N corners), Δ denotes the weak Laplacian on V and λ is a positive real parameter. In [6], Galewski has investigated the existence of at least two nontrivial weak solutions to a Dirichlet problem on the Sierpiński gasket. Some general abstract multiplicity theorems were developed which they were applied to the problem. The approach was based on the fact that the action functional is a difference of two continuously differentiable convex functionals and therefore the ideas related to the Fenchel-Young conjugacy together could be applied to get one critical point together with the mountain pass geometry to get the other one. In [10] Molica Bisci et al. have ensured the existence of at last two non-trivial (strong) solutions for the problem (1.1). In [8], the existence of multiple weak solutions for parametric quasi-linear systems of the gradient-type on the Sierpiński gasket was investigated. Some new criteria to guarantee that the systems have at least three weak solutions by using a variational method and some critical points theorems due to Ricceri were given.