

# Decoupling Analysis for Solving the Diffusive Peterlin Viscoelastic Model

Zhen Miao<sup>1</sup> and Yaolin Jiang<sup>2,\*</sup>

<sup>1</sup> School of Mathematics and Statistics, Northwestern Polytechnical University, Xi'an 710072, P.R. China

<sup>2</sup> School of Mathematics and Statistics, Xi'an Jiaotong University, Xi'an 710049, P.R. China

Received 14 November 2024; Accepted (in revised version) 20 May 2025

---

**Abstract.** The paper analyzes a system decoupling approach over the entire time domain to solve a diffusive Peterlin viscoelastic model which is a type of non-Newtonian fluids. The main idea is decoupling this non-Newtonian fluid equations into Newtonian fluid equations and second-order quasilinear parabolic equations entirely in the time domain, which can be solved iteratively. Under this approach, existing excellent computational method interfaces for Newtonian fluid equations and parabolic equations can be directly utilized. By employing the weak form of the Peterlin viscoelastic model, a specific decoupling scheme for the continuous-time case is developed. We prove the convergence of this decoupling scheme. Then, for the discrete-time case, a first-order scheme and its iterative error analysis are provided. Numerical experiments validate the theoretical convergence of the algorithm and its dependence on relative parameters.

**AMS subject classifications:** 65M12, 65M15, 76A05

**Key words:** Non-Newtonian fluid, decoupling approach, convergence, error analysis.

---

## 1. Introduction

Non-Newtonian fluids are a group of fluids that do not follow a linear relationship between the stress and deformation tensor. Based on molecular structures, non-Newtonian fluids exhibit various viscoelastic properties [20, 26]. The mainstream method for solving viscoelastic fluid problems can be referred to the review paper [2]. When solving time-dependent problems, the main approaches, such as the iterative-solution methods and fractional-step methods, are based on time-stepping techniques [4, 7, 21, 24]. Generally, serial numerical solutions of non-Newtonian fluid equations are expensive in terms of computational costs, which limits large-scale simulations of complex fluid flows.

---

\*Corresponding author. *Email address:* yljia@mail.xjtu.edu.cn (Y.-L. Jiang)

We are dedicated to proposing an approach to decoupling non-Newtonian fluid equations into Newtonian fluid equations. Thus, existing excellent computational method interfaces for Newtonian fluid equations can be directly utilized (such as using the popular projection methods, the energy, momentum and angular momentum conserving formulation, etc. [8, 10, 22]), which enhances the flexibility and ease of expansion of algorithms in large-scale computations. This approach comes from the idea of waveform relaxation methods, which were proposed due to the demand for high-performance computing of large-scale differential systems generated in circuit simulations. It has since undergone a lot of research on large ordinary differential equations, differential-algebraic equations, etc [12]. In terms of algorithms, they usually carried out through the overall calculation of time intervals and then iterating. Therefore, decoupling and simplifying systems are the most notable advantages. This allows for observing the waveform changes of each iteration and flexibly selecting time steps, and hence suitable for stiff systems. For semi-linear partial differential equations (PDEs), a kind of waveform relaxation at the PDE level can preserve the decoupling and parallel advantages and avoid deteriorating convergence caused by spatial mesh refinement [13]. However, for the decoupling of complex systems, it is necessary to ensure the convergence of iterative solutions, and the relevant theory is essential.

We study in this paper the decoupling approach for the diffusive Peterlin viscoelastic model, a recently studied non-Newtonian model. The Peterlin viscoelastic model follows the laws of conservation of mass and momentum, and a rheological constitutive equation that links velocity and non-Newtonian stress. For a given final time  $T > 0$  and external forces  $(f, \mathbf{F}) : \Omega \times (0, T] \rightarrow \mathbb{R}^2 \times \mathbb{R}^{2 \times 2}$ , three functions representing the fluid velocity  $u : \Omega \times (0, T] \rightarrow \mathbb{R}^2$ , the pressure  $p : \Omega \times (0, T] \rightarrow \mathbb{R}$ , and the symmetric positive definite conformation tensor  $\mathbf{C} : \Omega \times (0, T] \rightarrow \mathbb{R}_{\text{sym}}^{2 \times 2}$ , satisfy the following dimensionless equations in  $\Omega \times (0, T]$  [19]:

$$\begin{aligned} \partial_t u - \nu \Delta u + (u \nabla) u + \nabla p &= \nabla (\text{tr } \mathbf{C} \mathbf{C}) + f, \\ \nabla u &= 0, \\ \partial_t \mathbf{C} - \varepsilon \Delta \mathbf{C} + (u \nabla) \mathbf{C} - (\nabla u) \mathbf{C} - \mathbf{C} (\nabla u)^T &= \text{tr } \mathbf{C} \mathbf{I} + (\text{tr } \mathbf{C})^2 \mathbf{C} + \mathbf{F}, \end{aligned} \tag{1.1}$$

where  $\mathbf{I}$  is the identity matrix. The constants  $\nu, \varepsilon > 0$  correspond to fluid viscosity and elastic stress viscosity. We simply impose  $u(x, 0) = u_0(x)$  and  $\mathbf{C}(x, 0) = \mathbf{C}_0(x)$  in  $\Omega$ . Global existence and uniqueness result for the diffusive Peterlin viscoelastic model and more general situations in two dimensions were proved in [19].

Regarding the rigorous numerical analysis of computational methods, the linear and nonlinear stabilized Lagrange-Galerkin schemes for the Oseen-type Peterlin viscoelastic model were investigated in [17, 18]. These studies presented error estimates with optimal order convergence in two spatial dimensions. In [14], a semi-discrete Galerkin finite element spatial discretization was proposed and analyzed. Additionally, Ravindran [25] proposed a fully implicit stabilized Crank-Nicolson time-stepping scheme with finite element spatial discretization, and Han [9] proposed a decoupled second-order time-stepping scheme. In [30], a fully discrete two-grid finite element method