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Theoretical Guarantees of Recovery Algorithms for Ternary Sparse Signals

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Abstract. This paper focuses on the problem of recovering ternary sparse signals with s nonzero entries of 1 and -1. We propose three novel algorithms: ternary matching pursuit (TMP), ternary generalized orthogonal matching pursuit (TGOMP), and piecewise ternary generalized orthogonal matching pursuit (PTGOMP). First, inspired by the binary matching pursuit algorithm, we introduce the TMP algorithm, which assigns values of 1 or -1 based on the most correlated residual, and provide theoretical guarantees based on the mutual coherence, denoted by μ and the restricted isometry property of the measurement matrix, respectively. Second, we propose the TGOMP algorithm, by selecting multiple (M) indices at each iteration in order to improve the performance of the TMP algorithm. We establish a sufficient condition $\mu < 1/(2s-1)$ that ensures the TGOMP algorithm selects M correct indices and corresponding entries of $\boldsymbol{x}$ in each iteration. Especially, all correct indices and entries can be selected in the first iteration. Additionally, we present a sufficient condition based on the restricted isometry property that guarantees all correct indices are selected in at most s iterations. Third, we propose the PTGOMP algorithm, which employs a piecewise selection strategy at each iteration, further improves recovery performance. Theoretical guarantees for the PTGOMP algorithm are derived based on the mutual coherence, showing its advantages in ternary sparse signal recovery. Finally, we validate the effectiveness of our algorithms through simulations and numerical experiments, demonstrating that combining appropriate matrix structures with suitable sparsity patterns can significantly improve recovery performance.

AMS subject classifications: 94A12

Key words: Ternary sparse signal, mutual coherence, restricted isometry property, sparse recovery, piecewise sparsity.

1. Introduction

The problem of recovering sparse signals from an underdetermined noisy linear model has attracted significant attention [12, 13, 20, 44]. The model is typically for-

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mulated as

$$\boldsymbol{b} = A\boldsymbol{x} + \boldsymbol{v}, \tag{1.1}$$

where $\boldsymbol{b} \in \mathbb{R}^m$ denotes the observation vector, $A \in \mathbb{R}^{m\times n}$ denotes the measurement matrix with $m \ll n$, $\boldsymbol{x} \in \mathbb{R}^n$ is the s-sparse signal to be recovered (with s denoting the number of nonzero entries of $\boldsymbol{x}$), and $\boldsymbol{v} \in \mathbb{R}^m$ is additive noise with $\|\boldsymbol{v}\|_2 \le \epsilon$. Several algorithms have been proposed to achieve either exact or robust recovery of sparse signals, including basis pursuit (BP) [14], iterative hard thresholding (IHT) [7], compressive sampling matching pursuit (CoSaMP) [35], and orthogonal matching pursuit (OMP) [9, 25, 39], etc. Besides, a variant of the OMP algorithm, known as the generalized orthogonal matching pursuit (GOMP) [36,40] algorithm or the orthogonal multi-matching pursuit (OMMP) algorithm [17, 18, 45] has been widely studied.

The theoretical analysis of sparse recovery algorithms typically relies on two key concepts: the mutual coherence and the restricted isometry property (RIP). The mutual coherence of a matrix A is defined as [21]

$$\mu = \max_{i,j} \left|\langle \boldsymbol{a}_i, \boldsymbol{a}_j\rangle\right|,$$

where $\boldsymbol{a}_i$ and $\boldsymbol{a}_j$ are the unit-norm columns (or atoms) of the matrix A, for any $i \neq j$. Additionally, the matrix A satisfies the RIP [13] if there exists a constant $\delta \in (0,1)$ such that

$$(1-\delta)\|\boldsymbol{x}\|_2^2 \le \|A\boldsymbol{x}\|_2^2 \le (1+\delta)\|\boldsymbol{x}\|_2^2$$

for any s-sparse signal. The minimal value of δ is the restricted isometry constant (RIC), denoted δ_s.

Moreover, the inherent properties of sparse signals play an important role in the performance of recovery algorithms. Sparsity, the most widely studied property, is often referred to as global sparsity and is characterized by a single sparsity s. However, in many applications, such as magnetic resonance imaging (MRI) [24], computerized tomography (CT) [16], helium atom scattering [27], and fluorescence microscopy [37], signals exhibit additional structures that go beyond classical sparsity. Therefore, much research has focused on more complex forms of sparsity, including block sparsity [22, 23], joint sparsity [6, 8], and local sparsity in levels model [1, 2], also referred to as piecewise sparsity [30, 47].

Additionally, some sparse signals with certain magnitude and distribution of nonzero entries have been extensively studied. For instance, the binary sparse signals, where the nonzero entries are equal to 1, are applied in applications such as generalized space shift keying (GSSK) modulation detection [26] and massive connectivity detection in the Internet of things [15]. Algorithms such as the binary matching pursuit (BMP) [42] and the binary generalized orthogonal matching pursuit (BGOMP) [31] have been proposed to recover such signals. Similarly, the ternary sparse signals, where the nonzero entries are equal to 1 or -1, have been widely studied [28, 32–34, 38]. The ternary model is motivated by applications such as compressing neural networks [4, 19, 46], and the transmission of sparse signals [3, 5, 41]. The randomly aggregated