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The Method of Fundamental Solutions for Optical Fluorescence Tomography

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Abstract. In this paper, the method of fundamental solutions (MFS) is first developed for solving direct problems in bi-layer materials in the biomedical field of optical fluorescence. The governing system of second-order linear partial differential equations (PDEs) for the emission and excitation fluences is transformed into a single fourth-order PDE with appropriate boundary and interface matching conditions. The MFS is subsequently further developed, in conjunction with a constrained minimization regularization procedure, to solve nonlinear inverse optical fluorescence tomography problems. Numerical results confirm the accuracy, stability and versatility of the proposed meshless technique.

AMS subject classifications: Primary 65N35; Secondary 65N21, 65N38

Key words: Optical fluorescence, method of fundamental solutions, bi-layer materials, elliptic equations.

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1. Introduction

Optical fluorescence tomography (OFT) has emerged as an exciting molecular imaging lymph node or cancer detection technique. Through the injection of a fluorescent dye that directly targets specific tissues, OFT increases the signal-to-noise ratio detection when compared to the more well-known diffusive optical tomography (DOT) [2]. In several papers [2,14–17], Joshi and his co-workers developed a mathematical model for OFT governing the excitation light and the subsequent propagation of fluorescence light through a tissue medium [10].

In an earlier study [20], we proposed the method of fundamental solutions, as a suitable meshless method for solving the direct linear optical fluorescence boundary value problem (BVP). In the current paper, we advance the MFS to solving this BVP in

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a bi-layer material and the associated inverse nonlinear geometric problem. The latter requires the reconstruction of an unknown inclusion modelling an anomaly/defect that may be present within a biological tissue.

The plan of the paper is as follows. In Section 2, we introduce the governing optical fluorescence mathematical model based on a coupled system of two elliptic partial differential equations subjected to Robin third-kind convective boundary conditions (BCs) in a bi-layer material consisting of an anomaly surrounded by a healthy tissue. In Section 3, we introduce the MFS for a bi-material by extending our previous work [20] concerned with single-layer materials. Numerical experiments for the direct BVP of optical fluorescence in a bi-layer material are conducted in Section 4. In Section 5, the inverse geometric nonlinear OFT problem is formulated and solved numerically using the MFS combined with a constrained minimization technique whose objective function needs to be further regularized in order to achieve stability of the anomaly reconstruction. Finally, in Section 6 we highlight the conclusions of the paper.

2. Mathematical model

In non-dimensional form, the photon propagation in a bounded tissue Ω, in the frequency domain, is governed by the following system of PDEs [2, 16, 20]:

$$\Delta u - \alpha_{\mathrm{x}} u = 0 \quad \text{in } \Omega, \tag{2.1a}$$

$$\Delta v - \alpha_{\mathrm{m}} v = -B_{\mathrm{x}} u \quad \text{in } \Omega, \tag{2.1b}$$

where u and v are the excitation and emission light fluence, respectively,

$$\begin{aligned} \alpha_{\mathrm{x}} &= 3L^2(\mu_{\mathrm{axi}} + \mu_{\mathrm{axf}} + \mathrm{i}\omega/c)(\mu_{\mathrm{axi}} + \mu_{\mathrm{axf}} + \mu'_{\mathrm{sx}}), \\ \alpha_{\mathrm{m}} &= 3L^2(\mu_{\mathrm{ami}} + \mu_{\mathrm{amf}} + \mathrm{i}\omega/c)(\mu_{\mathrm{ami}} + \mu_{\mathrm{amf}} + \mu'_{\mathrm{sm}}), \\ B_{\mathrm{x}} &= \frac{3L^2\varphi\mu_{\mathrm{axf}}(\mu_{\mathrm{ami}} + \mu_{\mathrm{amf}} + \mu'_{\mathrm{sm}})}{1 - \mathrm{i}\omega\tau}, \end{aligned} \tag{2.2}$$

τ is the life time of the fluorosphore, φ is the probability of the re-emitted excitation fluence after dye absorption, L is a characteristic dimension of the spatial tissue domain, ω is the modulation frequency, $\mathrm{i} = \sqrt{-1}$, c is the speed of light, $\mu'_{\mathrm{sx}}, \mu'_{\mathrm{sm}}, \mu_{\mathrm{axi}}, \mu_{\mathrm{ami}}$ and $\mu_{\mathrm{axf}}, \mu_{\mathrm{amf}}$ are coefficients characterizing reduced scattering, absorption due to endogenous chromophores and absorption due to exogenous fluorosphore at the excitation and emission wavelengths, respectively.

The BCs associated to (2.1a) and (2.1b) are of Robin type and given by [2, 16, 20]

$$2\mathcal{D}_{\mathrm{x}} \frac{\partial u}{\partial n} + \gamma u + S = 0 \quad \text{on } \partial\Omega, \tag{2.3a}$$

$$2\mathcal{D}_{\mathrm{m}} \frac{\partial v}{\partial n} + \gamma v = 0 \quad \text{on } \partial\Omega, \tag{2.3b}$$