J. Partial Differential Equations
Vol. 5, *No.* 1 (1992), 69–78

ON THE SEMICONDUCTOR SYSTEM

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(Received Oct. 4, 1989; revised Aug. 29, 1990)

Abstract In this paper, the semiconductor system is discussed. The existence and uniqueness of the global solution of the carrier transport problem are obtained. Under the condition that the width in some direction of the domain being sufficiently small, the existence and uniqueness of the solution of the steady states are proved. It is also proved that the solution of the carrier transport problem tends to the solution of the steady states problem exponentially when t goes to infinity.
Key Words Semiconductor; strongly coupled system; upper-lower solution method; carrier transport; steady state; existence; uniqueness; asymptotic behaviour
Classification 35K57

Semiconductor system[1] is a semilinear partial differential system as follows

$$\begin{cases} \dfrac{1}{\mu_n} n_t = \Delta n - \nabla\cdot(n\nabla\phi) - R(n,p) \\ \dfrac{1}{\mu_p} p_t = \Delta p + \nabla\cdot(p\nabla\phi) - S(n,p) & \text{in } Q_T = \Omega\times[0,T) \\ \Delta\phi = 4\pi\eta(n - p - N_Y(x,t)) \end{cases} \tag{0.1}$$

The initial and boundary conditions are

$$\begin{cases} n|_{\partial\Omega} = \bar n(x,t), \quad p|_{\partial\Omega} = \bar p(x,t), \quad \phi|_{\partial\Omega} = \bar\phi(x,t) & \text{on } \partial\Omega\times[0,T) \\ n(x,0) = n_0(x), \quad p(x,0) = p_0(x) & \text{on } \bar\Omega \end{cases} \tag{0.2}$$

where $x = (x_1, x_2, \cdots, x_N), \Omega \subset R^N, N \geq 1; n, p$ are the densities of mobile holes and electrons respectively, ϕ is the electrostatic potential; $R(n,p) = r(n,p)(np-1)$, $S(n,p) = s(n,p)(np-1)$, $r(n,p)$ and $s(n,p)$ are positive Lip-continuous functions, $N_Y(x,t)$ is a positive smoothing function, η is a positive constant and

$$0 \leq r \leq \bar r, \quad 0 \leq s \leq \bar s, \quad 0 < N_Y < \bar N_Y \tag{0.3}$$

$$0 \leq \bar n, \bar p, n_0, p_0 \leq 1 \tag{0.4}$$

$$\bar{n},\bar{p},\bar{\phi}\in C^{2+\alpha,1+\frac{\alpha}{2}}(Q_T),\quad n_0,p_0\in C^{2+\alpha}(\Omega) \tag{0.5}$$

$$|\bar{n}-\bar{n}_\infty(x)|,\ |\bar{p}-\bar{p}_\infty(x)|,\ |\bar{\phi}-\bar{\phi}_\infty(x)|\le Ce^{-\gamma t},\quad \gamma>0 \tag{0.6}$$

These functions also satisfy the conditions of compatibility.

The steady states of this problem are defined as follows

$$\begin{cases} -\Delta n = -\nabla\cdot(n\nabla\phi) - R(n,p) \\ -\Delta p = \nabla\cdot(p\nabla\phi) - S(n,p) \qquad \text{in } \Omega \\ \Delta\phi = 4\pi\eta(n-p-N_Y) \end{cases} \tag{0.7}$$

$$n|_{\partial\Omega}=\bar{n}_\infty,\quad p|_{\partial\Omega}=\bar{p}_\infty,\quad \phi|_{\partial\Omega}=\bar{\phi}_\infty \tag{0.8}$$

They are strong coupled systems. Many people are interested in these two problems. There have been many works about them already in various ways and under different conditions, see [2]–[5].

In this paper, we use the upper-lower solution method[6],[7] to discuss these problems. We have the following results:

1. The existence and uniqueness of the global solution of the carrier transport problem (0.1), (0.2) under the conditions (0.3)–(0.5) are obtained. The solution is bounded and positive.

2. The existence and uniqueness of the solution of the steady states problem (0.7)–(0.8) under the condition of the domain Ω being sufficiently small in one direction are obtained. This condition means the matter of semiconductor is thin in one direction physically. In fact, P-N junction in semiconductor is very thin. The solution we get is also bounded and positive.

3. Under the condition of the domain Ω being sufficiently small in one direction and (0.6), the solution of the carrier transport problem (0.1)–(0.2) tends to the solution of the steady states problem (0.7)–(0.8) when t goes to infinity exponentially.

1. The Carrier Transport Problem

Consider a space

$$\mathcal{H}=L_\infty(0,T;L_q(\Omega)),\quad T>0,\quad 1<q<\infty$$

and a set

$$\mathcal{E}=\{(u,v)\in\mathcal{H}\,|\,0\le(u,v)\le e^{Mt}\}$$

where M will be determined later; $(a,b)\ \le,(\ge)C$ means $a\ \le,(\ge)C$ and $b\ \le,(\ge)C$; $(a,b)\le(c,d)$ means $a\le c, b\le d$.

Define a mapping on $\mathcal{H}, T(u,v) = (n,p)$: for $(u,v) \in \mathcal{E}, (n,p)$ is the solution of the problem as follows

$$\begin{cases} \dfrac{1}{\mu_n} n_t = \Delta n - \nabla n \cdot \nabla \phi - 4\pi\eta n(n-p-N_Y) - R(n,p) \\ \dfrac{1}{\mu_p} p_t = \Delta p + \nabla p \cdot \nabla \phi + 4\pi\eta p(n-p-N_Y) - S(n,p) \end{cases} \quad \text{in } Q_T \tag{1.1}$$

$$\Delta\phi = 4\pi\eta(u - v - N_Y) \quad \text{in } Q_T \tag{1.2}$$

with the initial-boundary condition (0.2).

At first, we solve ϕ from (1.2), (0.2), and we know that

$$\|\phi\|_{L_\infty(0,T;W_q^2(\Omega))} \le C_0(e^{MT}) \tag{1.3}$$

Then, the ϕ in (1.1) is known. Now, let us consider the problem (1.1) (0.2). The upper-lower solution method is used. According to the result in [6], the solution of the problem exists and is unique as long as there exists a pair of upper-lower solution. Thus, only one thing we need to do is to find out a pair of upper-lower solutions of the problem. Now, let us verify that $(\tilde{n}, \tilde{p}) = (e^{Mt}, e^{Mt}), (\underset{\sim}{n}, \underset{\sim}{p}) = (0,0)$ are just a pair of upper-lower solutions of the problem, where M will be determined later.

$$\begin{aligned} &\frac{1}{\mu_n}\tilde{n}_t - \Delta\tilde{n} + \nabla\tilde{n}\cdot\nabla\phi - \sup_{\underset{\sim}{p} \le \eta \le \tilde{p}} \{-4\pi\eta\tilde{n}(\tilde{n} - \eta - N_Y) - R(\tilde{n}, \eta)\} \\ &\ge \frac{1}{\mu_n} M e^{Mt} + 4\pi\eta e^{Mt}(e^{Mt} - e^{Mt} - N_Y) - r(e^{Mt}, \eta)(e^{Mt} \times 0 - 1) \\ &\ge e^{Mt}\Big(\frac{M}{\mu_n} - 4\pi\eta\bar{N}_Y - \bar{r}\Big) \ge 0 \end{aligned} \tag{1.4}$$

as long as

$$M \ge 4\pi\eta\bar{N}_Y\mu_n + \bar{r}\mu_n + \bar{s}\mu_p \tag{1.5}$$

And $\tilde{p} = e^{Mt}$ is almost the same to be verified in equation as (1.4).

On the boundary, we have (0.4). That is, (e^{Mt}, e^{Mt}) is satisfied to be an upper solution of the problem, as long as (1.5) holds.

$$\begin{aligned} &\frac{1}{\mu_n} n_t - \Delta n + \nabla n \cdot \nabla\phi - \inf_{\underset{\sim}{p} \le \eta \le \tilde{p}} \{-4\pi\eta n(n - \eta - N_Y) - R(n, \eta)\} \\ &= -r(0, \eta) \le 0. \end{aligned} \tag{1.6}$$

And $\underset{\sim}{p} = 0$ is almost the same to be verified in equation as (1.6). We also have (0.4) on the boundary, then (0, 0) is satisfied as a lower solution of the problem.

In addition, it is obvious that the reaction terms of the problem are continuous and bounded by the upper and lower solutions. Thus, from [6], we know that the problem

(1.1) (0.2) has a unique solution (n,p) in Q_T, and it belongs to $W_q^{2,1}(Q_T), 1 < q < \infty, 0 \le (n,p) \le e^{Mt}$. Thus

$$T(\mathcal{E}) \subset \mathcal{E} \quad \text{(compact)}$$

Then the continuity of T is not difficult to verified. Set $T(u_i, v_i) = (n_i, p_i), i = 1, 2, \phi_i$ is the solution of the problem

$$\begin{cases} \Delta \phi_i = 4\pi\eta(u_i - v_i - N_Y) \\ \phi_i|_{\partial\Omega} = \bar{\phi}(x,t) \end{cases}$$

Now let $(w, z, \psi) = (n_1 - n_2, p_1 - p_2, \phi_1 - \phi_2)$, from the equations and the boundary-initial conditions we can get the following

$$\begin{aligned} &\|w\|^q_{L_q(\Omega)} + \|z\|^q_{L_q(\Omega)} \\ &\le C(\|u_1 - u_2\|^q_{\mathcal{X}} + \|v_1 - v_2\|^q_{\mathcal{X}} + \int_0^t (\|w\|^q_{L_q(\Omega)} + \|z\|^q_{L_q(\Omega)})dt) \end{aligned} \quad (1.7)$$

By Gronwall inequality, now we get the estimate of continuity of T

$$\|w\|^q_{\mathcal{X}} + \|z\|^q_{\mathcal{X}} \le C(\|u_1 - u_2\|^q_{\mathcal{X}} + \|v_1 - v_2\|^q_{\mathcal{X}}) \quad (1.8)$$

Then by Schauder's Fixed Point Theorem[10], the mapping T has at least one fixed point. Therefore, the problem (0.1)–(0.2) has a solution.

Now, let us prove the uniqueness of the solution.

If there were two solutions (n_1, p_1, ϕ_1) and (n_2, p_2, ϕ_2), let us set $(w, z, \psi) = (n_1 - n_2, p_1 - p_2, \phi_1 - \phi_2)$, then we can obtain the estimate

$$\|w\|^q_{L_q(\Omega)} + \|z\|^q_{L_q(\Omega)} \le C \int_0^t (\|w\|^q_{L_q(\Omega)} + \|z\|^q_{L_q(\Omega)})dt$$

By Gronwall inequality, we have

$$0 \le \|w\|^q_{\mathcal{X}} + \|z\|^q_{\mathcal{X}} = 0 \quad (1.9)$$

That is, the solution is unique.

By Schauder's smoothing result [10], we know that $n, p \in C^{2+\alpha, 1+\frac{\alpha}{2}}(Q_T)$, then $\phi \in C^{2+\alpha, 1+\frac{\alpha}{2}}(Q_T)$. Thus, we have

Theorem 1 *Under the conditions* (0.3)–(0.5), *the solution of the problem* (0.1), (0.2) *exists and is unique. And the solution satisfies*

$$0 \le (n,p) \le e^{Mt}, \quad |\phi| \le C(e^{Mt}), \quad n, p, \phi \in C^{2+\alpha, 1+\frac{\alpha}{2}}(Q_T)$$

2. The steady state Problem

Throughout this section, we suppose that the domain Ω is sufficiently small in one direction, let us say this direction is $y : \Omega \subset \Omega' \times (0, y_0)$, and y_0 is small.

Consider a space $\mathcal{H} = L_\infty(\Omega)$ and a set $\mathcal{E} = \{(u,v) \in \mathcal{H} | 0 \leq (u,v) \leq B - Ay^2\}$, where A, B will be determined later.

Define a mapping T on $\mathcal{E}$, $T(u,v) = (n,p)$: for $(u,v) \in \mathcal{E}, (n,p)$ is the solution of the problem as follows

$$\begin{cases} -\Delta n = -\nabla n \cdot \nabla \phi - 4\pi\eta n(n - v - N_Y) - R(n,v) \\ -\Delta p = \nabla p \cdot \nabla \phi + 4\pi\eta p(u - p - N_Y) - S(u,p) \end{cases} \tag{2.1}$$

$$\Delta \phi = 4\pi\eta(u - v - N_Y) \tag{2.2}$$

with the boundary condition (0.8).

We can find out ϕ, from (2.2) first, then we can consider the problem of (2.1), (0.8). This problem has two uncoupled equations, and $(\tilde{n}, \tilde{p}) = (B - Ay^2, B - Ay^2), (\underset{\sim}{n}, \underset{\sim}{p}) = (0,0)$ are the upper and lower solutions respectively. Let us verify them

$$\begin{aligned} -\Delta\tilde{n} + \nabla\tilde{n} \cdot \nabla\phi - \sup_{\underset{\sim}{p} \leq \eta \leq \tilde{p}} \{-4\pi\eta\tilde{n}(\tilde{n} - \eta - N_Y) - R(\tilde{n}, \eta)\} \\ \geq 2A - 2Ay(C'\|u - v - N_Y\|_0 + 1) - 4\pi\eta B\bar{N}_Y - \bar{r} \\ \geq 2A - 2Ay[C'(B + \bar{N}_Y) + 1] - 4\pi\eta B\bar{N}_Y - \bar{r} \geq 0 \end{aligned}$$

as long as

$$y_0 \leq \frac{1}{2[C'(B + \bar{N}_Y) + 1]}, \quad A \geq 4\pi\eta B\bar{N}_Y + \bar{r} + \bar{s}, \quad B \geq \frac{4\pi\eta B\bar{N}_Y + \bar{r} + \bar{s}}{4[C'(B + \bar{N}_Y) + 1]^2} + 1 \tag{2.3}$$

where if B is taken big enough, the third inequality will be satisfied. In the above, the following is used

$$\begin{aligned} \|\nabla\phi\|_{L_\infty} &\leq C\|\Delta\phi\|_{L_\infty} + \|\phi\|_{L_\infty} \\ &\leq C'\|u - v - N_Y\|_{L_\infty} + \|\bar{\phi}\|_{L_\infty} \leq C'(B + \bar{N}_Y) + 1 \end{aligned}$$

In almost the same way for $\tilde{p} = B - Ay^2$, as long as (2.3) holds. And on the boundary,

$$(B - Ay^2, B - Ay^2) \geq (1,1) \geq (\bar{n}_\infty, \bar{p}_\infty)$$

as long as (2.3) holds.

Then, $(B - Ay^2, B - Ay^2)$ is just an upper solution of the problem. And it is easy to verify that (0, 0) is a lower solution of the problem.

From the equation's upper-lower solution theory, see the problem (2.1) (2.2) and (0.8) of Chap.10.B in [7] has the solution (n,p), and

$$n,p \in C^{2+\alpha}(\Omega); \quad 0 \le (n,p) \le B - Ay^2$$

That is,

$$T(\mathcal{E}) \subset \mathcal{E} \quad \text{(compact)}$$

And it is not difficult to get continuity of T. We can obtain the following estimate

$$\|w\|_{L_\infty} + \|z\|_{L_\infty} \le C(\|u_1 - u_2\|_{L_\infty} + \|v_1 - v_2\|_{L_\infty})$$

Then, by Schauder's Fixed Point Theorem[10], the mapping T has at least one fixed point. Therefore, the solution of the problem (0.7)–(0.8) exists.

Now, let us discuss the uniqueness of the problem (0.7)–(0.8). If there were two solutions: $(n_i, p_i, \phi_i), i = 1,2$, we set $w = n_1 - n_2, z = p_1 - p_2, \psi = \phi_1 - \phi_2$, then, w, z, ϕ satisfy

$$\begin{cases} -\Delta w = -\nabla\cdot(w\nabla\phi_1) - \nabla\cdot(n_2\nabla\psi) - (R(n_1,p_1) - R(n_2,p_2)) \\ -\Delta z = \nabla\cdot(z\nabla\phi_1) + \nabla\cdot(p_2\nabla\psi) - (S(n_1,p_1) - S(n_2,p_2)) \\ \Delta\psi = 4\pi\eta(w - z) \\ w|_{\partial\Omega} = z|_{\partial\Omega} = \psi|_{\partial\Omega} = 0 \end{cases} \tag{2.4}$$

multiplying the first two equations of (2.4) by w and z respectively, and integrating by parts on Ω, we have

$$\int_\Omega (|\nabla w|^2 + |\nabla z|^2)dx \le \bar{C}\int_\Omega (w^2 + z^2)dx \tag{2.5}$$

where $\bar{C}$ depends on B and independent of y_0. Note that

$$\int_\Omega w^2 dx = \int_\Omega \left(\int_0^y w_y\right)^2 dx \le y_0 \int_\Omega |\nabla w|^2 dx, \quad \int_\Omega z^2 dx \le y_0 \int_\Omega \nabla z^{\ 2} dx$$

As long as y_0 be small enough, such that

$$\bar{C}_{y_0} \le \frac{1}{2}$$

Then

$$\int_\Omega (|\nabla w|^2 + |\nabla z|^2)dx \le 0 \tag{2.6}$$

Hence, $|\nabla w|, |\nabla z| = 0$, a.e. So, $w, z =$const, but $w|_{\partial\Omega} = z|_{\partial\Omega} = 0$. Then, $w = z = 0$, and $\psi = 0$.

That is, the solution is unique, as long as y_0 is small enough.

From all above, we have

Theorem 2 *Under the condition of the domain Ω being sufficiently small in one direction, the solution of the problem* (0.7)–(0.8) *exists and is unique. The solution also satisfies that*

$$0 \le (n,p) \le B; \quad |\phi| \le C(B); \quad n,p,\phi \in C^{2+\alpha}(\Omega)$$

3. Asymptotic Behaviour

In this section, we also suppose that y_0 is small enough.

At first, we need to get the bound of the solution of the carrier transport problem (0.1)–(0.2) which is independent of t.

Verify that $(B - Ay^2, B - Ay^2)$, $(0, 0)$ are a pair of upper-lower solution of the problem (1.1)–(0.2), in the set $\mathcal{E} = \{(u,v) \in \mathcal{H} | 0 \le (u,v) \le B - Ay^2\}$, for example, for equation of n:

$$\begin{aligned}&\frac{1}{\mu_n}\tilde{n}_t - \Delta\tilde{n} + \nabla\tilde{n}\cdot\nabla\phi - \sup_{\underset{\sim}{p} \le \eta \le \tilde{p}} \{-4\pi\eta\tilde{n}(\tilde{n} - \eta - N_Y) - R(\tilde{n},\eta)\}\\ &\ge 2A - 2Ay(C'\|u - v - N_Y\|_0 + 1) - 4\pi\eta B\bar{N}_Y - \bar{r} \ge 0\end{aligned}$$

as long as (2.3) holds and for the p equation as well as boundary-initial condition can be done as before.

That $(0, 0)$ is a lower solution can be verified easily, too.

Thus, replace $B - Ay^2$ to e^{Mt} in the proof of Theorem 1, we can derive that

Theorem 3 *If the domain Ω is small enough in one direction, and* (0.3)–(0.5) *are satisfied, the solution of the carrier transport problem* (0.1)–(0.2) *has the estimate independent of t:*

$$0 \le (n,p) \le B, \quad |\phi| \le C(B) \tag{3.1}$$

In the following, we will denote the solution of steady states problem by (n^*, p^*, ϕ^*) to distinguish the (n,p,ϕ), one of the solution of the carrier transport problem.

Consider an auxiliary problem

$$\begin{cases}\left.\begin{aligned}&\frac{1}{\mu_n}\hat{n}_t = \Delta\hat{n} - \nabla\hat{n}\cdot\nabla\phi^* - 4\pi\eta n^*(n^* - p^* - N_Y) - R(n^*,p^*)\\ &\frac{1}{\mu_p}\hat{p}_t = \Delta\hat{p} - \nabla\hat{p}\cdot\nabla\phi^* + 4\pi\eta p^*(n^* - p^* - N_Y) - S(n^*,p^*)\end{aligned}\right\} \text{ in } Q_T\\ \hat{n}|_{\partial\Omega} = \bar{n}, \quad \hat{p}|_{\partial\Omega} = \bar{p} \qquad \text{on } \partial\Omega\times[0,T]\\ \hat{n}|_{t=0} = n_0, \quad \hat{p}|_{t=0} = p_0 \qquad \text{on } \Omega\end{cases} \tag{3.2}$$

Lemma *The problem* (3.2) *has a unique solution, and this solution has the estimate*

$$|\hat{n} - n^*| \le e^{-\delta t}g, \quad |\hat{p} - p^*| \le e^{-\delta t}g \tag{3.3}$$

where δ is a positive constant and will be determined later, $g = B - Ay^2$.

Proof Verify that $(\tilde{n},\tilde{p}) = (n^* + e^{-\delta t}g, p^* + e^{-\delta t}g)$, $(\underset{\sim}{n},\underset{\sim}{p}) = (n^* - e^{-\delta t}g, p^* - e^{-\delta t}g)$ are a pair of upper-lower solution of the problem (3.2).

At first, with the condition (0.6), on the boundary, we have

$$(n^*+e^{-\delta t}g,p^*+e^{-\delta t}g)|_{\partial\Omega}\geq(\bar{n}_\infty+e^{-\gamma t},\bar{p}_\infty+e^{-\gamma t})\geq(\tilde{n},\tilde{p})$$
$$\geq(\bar{n}_\infty-e^{-\gamma t},\bar{n}_\infty-e^{-\gamma t})\geq(n^*-e^{-\delta t}g,p^*-e^{-\delta t}g)|_{\partial\Omega}$$

$$(n^*+e^{-\delta t}g,p^*+e^{-\delta t}g)|_{t=0}\geq(\bar{n}_\infty+1,\bar{p}_\infty+1)\geq(n_0,p_0)$$
$$\geq(\bar{n}_\infty-1,\bar{p}_\infty-1)\geq(n^*-e^{-\delta t}g,p^*-e^{-\delta t}g)|_{t=0}$$

as long as $\delta<\gamma$.

Then let us see the equation

$$\frac{1}{\mu_n}\tilde{n}_t-\Delta\tilde{n}+\nabla\tilde{n}\cdot\nabla\phi^*+4\pi\eta n^*(n^*-p^*-N_Y)+R(n^*,p^*)$$
$$\geq-(\Delta n^*-\nabla n^*\cdot\nabla\phi^*-4\pi\eta n^*(n^*-p^*-N_Y)-R(n^*,p^*))$$
$$+e^{-\delta t}\Big(-\frac{\delta g}{\mu_n}+2A-2Ay(C'\|u^*-v^*-N_Y\|_0+1)\Big)$$
$$\geq e^{-\delta t}\Big(-\frac{\delta g}{\mu_n}+2A-2Ay_0(C'(B+\bar{N}_Y)+1)\Big)\geq 0$$

as long as

$$y_0\leq\frac{1}{2[C'(B+\bar{N}_Y)+1]},\quad \delta=\min\Big\{\gamma,\frac{A\min\{\mu_n,\mu_p\}}{B}\Big\}\tag{3.4}$$

and it is almost the same for p equation as long as (3.4) holds.

That is, $(\tilde{n},\tilde{p})$ is an upper solution. The lower solution can be verified in almost the same way as long as (3.3) holds.

We can note that if y_0 is smaller, δ can be take a little bigger, then the speed of the convergence will be greater.

Subtracting (3.2) from (1.1) and set $(w,z)=(n-\hat{n},p-\hat{p})$, then, w and z satisfy

$$\begin{cases}\dfrac{1}{\mu_n}w_t-\Delta w+\nabla w\cdot\nabla\phi^*\\ \quad=-\nabla(\phi-\phi^*)\cdot\nabla n-4\pi\eta(n-n^*)(n^*-p^*-N_Y)\\ \quad\quad-4\pi\eta n^*((n-n^*)-(p-p^*))-(R(n,p)-R(n^*,p^*))\\ \dfrac{1}{\mu_p}z_t-\Delta z-\nabla z\cdot\nabla\phi^*\\ \quad=\nabla(\phi-\phi^*)\cdot\nabla n+4\pi\eta(p-p^*)(n^*-p^*-N_Y)\\ \quad\quad+4\pi\eta p^*((n-n^*)-(p-p^*))-(S(n,p)-S(n^*,p^*))\\ \Delta(\phi-\phi^*)=4\pi\eta((n-n^*)-(p-p^*))\\ w|_{\partial\Omega}=z|_{\partial\Omega}=0,\quad(\phi-\phi^*)|_{\partial\Omega}=\bar{\phi}-\bar{\phi}_\infty,\quad w(x,0)=z(x,0)=0\end{cases}\tag{3.5}$$

The existence and uniqueness of this problem is obvious. By L_p Estimate[8] and (3.3), we have

$$\begin{aligned}\|w\|_{W_q^{2,1}(Q_T)}+\|z\|_{W_q^{2,1}(Q_T)} &\le C(\|n-n^*\|_{L_q(Q_T)}+\|p-p^*\|_{L_q(Q_T)})\\ &\le C(\|w\|_{L_q(Q_T)}+\|z\|_{L_q(Q_T)}+\|\hat{n}-n^*\|_{L_q(Q_T)}+\|\hat{p}-p^*\|_{L_q(Q_T)})\\ &\le Cy_0(\|w\|_{W_q^{2,1}(Q_T)}+\|z\|_{W_q^{2,1}(Q_T)})+Ce^{-\delta t}\end{aligned}$$

For $y_0 \le \dfrac{1}{2C}$, we have

$$\|w\|_{W_q^{2,1}(Q_T)}+\|z\|_{W_q^{2,1}(Q_T)} \le Ce^{-\delta t}$$

So,

$$\|w\|_{L_\infty}+\|z\|_{L_\infty} \le Ce^{-\delta t}$$

Then,

$$\|n-n^*\|_{L_\infty} \le \|n-\hat{n}\|_{L_\infty}+\|\hat{n}-n^*\|_{L_\infty} \le Ce^{-\delta t}, \quad \|p-p^*\|_{L_\infty} \le Ce^{-\delta t}$$

$$\|\phi-\phi^*\|_{L_\infty} \le C(\|(n-n^*)-(p-p^*)\|_{L_\infty}+\|\bar{\phi}-\bar{\phi}_\infty\|_{L_\infty}) \le 2Ce^{-\delta t}$$

Now, we have

Theorem 4 *If the domain Ω is small in one direction and* (0.6) *is satisfied, the solution (n,p,ϕ) of the carrier transport problem* (0.1)–(0.2) *tends to the solution (n^*,p^*,ϕ^*) of the steady states problem in exponent when t tends to infinite.*

In my working on this paper, Prof. Ye Qixiao has given me some helpful advices. Here, let me express my gratitude to him.

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