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NUMERICAL INVERSION OF MULTIDIMENSIONAL LAPLACE TRANSFORMS USING MOMENT METHODS*

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Abstract *This paper develops a numerical method to invert multi-dimensional Laplace transforms. By a variable transform, Laplace transforms are converted to multi-dimensional Hausdorff moment problems so that the numerical solution can be achieved. Stability estimation is also obtained. Numerical simulations show the efficiency and practicality of the method.*

Key words *Numerical Inversion of Laplace Transform, ill-posed Problems, multi-dimension, moment Problem, numerical computation*

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1 Introduction

The numerical inversion of Laplace transform function is sometimes necessary in the solution of engineering problem. Simple functions can be treated with transform tables. Software packages, such as MATLAB, MAPLE and MATHEMATICA, allow the user to obtain the analytical transform and the inverse. However complicated transform functions arise which are difficult to invert by analytical methods. Several efficient numerical inverse Laplace transform methods have been developed for various classes of functions[1−5] and we should say there is no 'best' method[6]. A moment problem from the Laplace transform[7] is studied for one-dimensional case. In this paper, we shall develop a numerical method to invert multi-dimensional Laplace transforms based on Hasdorff moment problems[8−10].

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The one-dimensional Laplace transform of a real-valued function $f(t)$, is given by an integral where s is a complex variable $s=\alpha+i\omega$ known as the Laplace variable,

$$F(s)=\int_0^\infty f(t)e^{-st}\mathrm{d}t, \tag{1}$$

There are two restrictions on $f(t)$ that are sufficient for the integral to exist:

(i) $f(t)$ must be piecewise continuous;

(ii) $f(t)$ must be of exponential order: $|f(t)|<Me^{\alpha t}$.

In the numerical method of this paper, we need select a value of α such that the second restriction is satisfied.

We are interested in multidimensional transform inversion in this paper. For simplicity, we consider only the bivariate case, but the algorithm extends directly to n-dimensional functions. Thus, our goal is to calculate values of a real-valued function f defined on the positive quadrant of the plane $\mathbb{R}_+^2\equiv[0,\infty)\times[0,\infty)$ from finite measurements, by numerically inverting its Laplace transform

$$F(s_1,s_2)=\int_0^\infty\int_0^\infty e^{-(s_1t_1+s_2t_2)}f(t_1,t_2)\mathrm{d}t_1\mathrm{d}t_2 \tag{2}$$

which we assume is well defined, e.g., convergent and thus analytic for $Re(s_1)>\alpha_1$ and $Re(s_2)>\alpha_2$, where $|f(t_1,t_2)|<Me^{\alpha_1t_1+\alpha_2t_2},\forall t_1,t_2\in\mathbb{R}_+$, M is a positive constant.

2 Transformation of multidimensional Laplace transform

In this paper, we consider to invert $f(t_1,t_2)$ numerically by its Laplace transform (2) from the finite measurements $F(s_{1i},s_{2j})$, where $i=1,2,\cdots,N_1, j=1,2,\cdots,N_2$, $N_1,N_2\in\mathbb{Z}_+$.

Let $x_1=e^{-t_1}, x_2=e^{-t_2}$. Then the Laplace transform (2) can be written after some computations

$$F(s_1,s_2)=\int_0^\infty\int_0^\infty x_1^{s_1-1-\alpha_1}x_2^{s_2-1-\alpha_2}u(x_1,x_2)\mathrm{d}x_1\mathrm{d}x_2 \tag{3}$$

where $u(x_1,x_2)=x_1^{\alpha_1}x_2^{\alpha_2}f(-\ln(x_1),-\ln(x_2))$, or $f(t_1,t_2)=e^{\alpha_1t_1}e^{\alpha_2t_2}u(e^{-t_1},e^{-t_2})$. For $s_1=m+1+\alpha_1, s_2=n+1+\alpha_2$, equation (3) becomes

$$\int_0^\infty\int_0^\infty x_1^m x_2^n u(x_1,x_2)\mathrm{d}x_1\mathrm{d}x_2=\mu_{mn}, \tag{4}$$

where $m,n=0,1,2,\cdots,\mu_{mn}=F(m+1+\alpha_1,n+1+\alpha_2)$. Problem (4) is a two-dimensional Hausdorff moment problem and we can use the method of paper [8,10] to regularize it. Then the numerical inversion of Laplace transform (2) is equivalent to the two-dimensional Hausdorff moment problem.