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AN ASYMPTOTIC PRESERVING IMPLICIT UNIFIED GAS KINETIC SCHEME FOR FREQUENCY-DEPENDENT RADIATIVE TRANSFER EQUATIONS

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Abstract. In this paper, an asymptotic preserving implicit unified gas kinetic scheme (IUGKS) is constructed for the frequency-dependent radiative transfer equations. Different from the asymptotic preserving unified gas kinetic scheme (UGKS) which uses the explicit initial value of the radiation intensity in the construction of the boundary fluxes as in the previous works [Sun et al., J. Comput. Phys. 285 (2015), pp. 265-279 and J. Comput. Phys. 302 (2015), pp. 222-238], here we construct the boundary fluxes by a back-time discretization so that they depend implicitly on the radiation intensity. Thus, the time step constraint by the Courant-Friedrichs-Lewy (CFL) condition is not needed anymore for IUGKS. It is shown that IUGKS is asymptotic preserving uniformly with the small Knudsen parameter. A number of numerical tests have been carried out and the numerical results show that large time steps can be used for the current scheme, and the computational efficiency can be improved greatly in comparison with UGKS and the implicit Monte Carlo scheme.

Key words. radiative transfer, frequency-dependent, asymptotic preserving, implicit unified gas kinetic scheme(IUGKS).

1. Introduction

Numerical solution of the radiative transfer equations is very important in many research fields, such as in astrophysics, inertial confinement fusion, and high temperature flow systems. Due to the complexity and higher dimensionality of the system, the numerical solution of the radiative transfer equations is very challenging, and its study attracts continuous attention from national laboratories and academic institutes. In this paper we shall make a continuous effort to develop a useful and reliable computational method for multiple scale radiative transfer systems.

The radiative transfer equations are the modeling equations in the kinetic level, where the photon transport and collision with material are taken into account. This system can present different limiting solutions with the changing of the scales. For the gray radiative transfer equations, the opacity is just a function of the material temperature. Therefore, the spatial cells can be classified as optical thick and optical thin regions, and a domain decomposition method with different numerical discretization in different regions can be developed. However, for the frequency-dependent radiative transfer equations, the opacity is typically a decreasing function of frequency. A spatial region can be optically thick for low frequency photon, but optically thin for high frequency ones.

Since the radiative transfer equations model the radiation intensity transport and energy exchange with the background material. The properties of the background material influence greatly on the behavior of radiation transfer. For a low opacity (background) material, the interaction between the radiation and material is weak, and the radiation propagates in a transparent way. The numerical method in this regime can be well developed by tracking the particle streaming transport.

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However, for a high opacity (background) material, there is severe interaction between radiation and material with a diminishing photon mean free path. As a result, the diffusive radiative behavior will dominate. In order to solve the kinetic scale based radiative transfer equations numerically, a straightforward way is to use a spatial mesh size which is comparable with photon's mean-free path, i.e., the so-called optical thin cell, and the transport equation can be discretized directly, such as using upwind approach for photon transport. This kind of method is basically a single scale method, where the numerical resolution down to the mean free path is used everywhere in the computation. Most Monte Carlo methods for transport equations belong to this category as well. In this kind of methods, to take such a small cell size will be associated with huge computational costs in the optical thick regime. In order to use a large cell size in comparison with the mean free path in the optical thick region, instead of decoupling the particle transport and collision in the numerical discretization, the coupled transport and collision has to be taken into account in the design of the scheme.

One of the efficient multiscale methods is to develop the so-called asymptotic preserving (AP) scheme for the kinetic equation. When holding the mesh size and time step fixed and as the Knudsen number going to zero, the AP scheme should automatically recover the discrete diffusion solution. AP schemes were first studied in the numerical solution of steady neutron transport problems by Larsen, Morel and Miller [17], Larsen and Morel [16], and then by Jin and Levermore [10,11], and the others. For unsteady problems, one of the AP schemes was constructed based on a decomposition of the distribution function between an equilibrium part and its non-equilibrium derivation, see Klar [13,14], and Jin, Pareschi and Toscani [12] for details. The development of an AP-type discrete ordinate method (DOM) for the multi-frequency radiative transfer equation coupled with material energy equation is a challenging numerical problem [8, 21], where most well-validated approaches are the Monte Carlo methods.

The unified gas kinetic scheme (UGKS) is one of the AP schemes for the transport equations [9, 22, 24, 26, 27]. It not only recovers accurate limiting solutions, such as ballistic transport and diffusion propagation, but also presents reliable solution in the whole transition regime. In UGKS, the mesh size is used directly as a modeling scale for identifying transport dynamics. When the mesh size is on the order of mean free path, the kinetic transport mechanism, such as the modeling process of the Boltzmann equation, is recovered in the numerical evolution [25]. When the mesh size is much larger than the mean free path, the hydrodynamic scale physics, such as the Navier-Stokes (NS) solutions for the flow system and the diffusion equation for the radiative transfer, is obtained. Between these two limits, a smooth transition is constructed and used for the capturing of non-equilibrium phenomena. In UGKS the mesh size and time step are dynamic variables in the evolution model. It may not be difficult to accept this kind of concept if we can realize that all fluid dynamic equations, such as the Boltzmann equation and the NS equations, are constructed based on their specific modeling scales with the corresponding dynamics.

In the previous works, we have developed an asymptotic preserving UGKS for the gray radiative transfer equations [22], and then an extension to frequency-dependent radiative transfer system [27]. Because the reconstruction method for the boundary fluxes in ([22], [27]) is explicit with the initial value of the radiative intensity, the time step should be constrained by the Courant-Friedrichs-Lewy (CFL) condition. The computational times could be very large for small spatial