Commun. Comput. Phys.
doi: 10.4208/cicp.2014.m303

Vol. **17**, No. 4, pp. 908-924
April 2015

A Localized Mass-Conserving Lattice Boltzmann Approach for Non-Newtonian Fluid Flows

Liang Wang[1,2], Jianchun Mi[1], Xuhui Meng[2] and Zhaoli Guo[2,*]

[1] *State Key Laboratory of Turbulence and Complex Systems, College of Engineering, Peking University, Beijing 100871, P.R. China.*
[2] *State Key Laboratory of Coal Combustion, Huazhong University of Science and Technology, Wuhan, 430074, P.R. China.*

Received 4 October 2013; Accepted (in revised version) 4 April 2014

Abstract. A mass-conserving lattice Boltzmann model based on the Bhatnagar-Gross-Krook (BGK) model is proposed for non-Newtonian fluid flows. The equilibrium distribution function includes the local shear rate related with the viscosity and a variable parameter changing with the shear rate. With the additional parameter, the relaxation time in the collision can be fixed invariable to the viscosity. Through the Chapman-Enskog analysis, the macroscopic equations can be recovered from the present mass-conserving model. Two flow problems are simulated to validate the present model with a local computing scheme for the shear rate, and good agreement with analytical solutions and/or other published results are obtained. The results also indicate that the present modified model is more applicable to practical non-Newtonian fluid flows owing to its better accuracy and more robustness than previous methods.

PACS: 44.05.+e, 47.11.-j, 47.56.+r

Key words: Lattice Boltzmann method, non-Newtonian fluids, mass conservation.

1 Introduction

Non-Newtonian fluid flows play an important role in many industrial and natural processes such as casting of concrete, mould filling and slides, and drilling mud pumping. From an analytical standpoint, the constitutive behaviour of non-Newtonian fluid is generally nonlinear, and it is hard to find analytical solutions for the related flows. Therefore, efficient numerical methods are highly sought for studying non-trivial flow behaviours of non-Newtonian fluid in many cases of interest. Different from Newtonian fluids, the

*Corresponding author. *Email addresses:* wlsa0612@gmail.com (L. Wang), jcmi@coe.pku.edu.cn (J. C. Mi), mengxuhui@hust.edu.cn (X. H. Meng), zlguo@hust.edu.cn (Z. L. Guo)

viscosity of non-Newtonian fluids is not constant and depends on the local shear rate or the gradient of velocity. Accurate and efficient methods to evaluate spatial velocity derivatives are therefore required for numerical approaches. As a mesoscopic numerical approach, an advantage of the lattice Boltzmann method (LBM) for non-Newtonian fluid is that the shear tensor can be computed locally without troublesome computing the velocity gradient [1] while the computation has second-order accuracy in space [2,3]. In the past decades, the LBM has been applied to non-Newtonian flow systems [4–13].

In the lattice Boltzmann model for non-Newtonian fluid flows, the simplest one is the Bhatnagar-Gross-Krook (BGK) model on the basis of a single-relaxation-time approximation [14, 15]. Owing to its simplicity, the BGK model has become the most popular one. However, in most of the BGK-based models, the local viscosity of non-Newtonian fluid is determined by the relaxation time (λ) which varies with the shear rate at each step. As λ approaches to 0.5 (the small-viscosity limit), these models often suffer from numerical instability [16, 17]. To overcome this deficiency, several modifications have been made. For instance, Gabbanelli et al. [18] imposed lower and upper bounds on the fluid viscosity to avoid numerical instability. The same truncated treatment on the viscosity of power-law fluid was also adopted by Pontrelli et al. [19]. Obviously, this artificial treatment will give rise to limited applications in the range of shear-dependent viscosities. Later, the lattice scheme proposed by Inamuro [20] for Newtonian viscous fluid flows was extended to non-Newtonian fluid flows by Yoshino et al. [21], in which the relaxation time is set to be unity while an additional term is incorporated into the equilibrium distribution function to represent the fluid viscosity. In such a way, the fluid viscosity is determined by another parameter independent with the relaxation time, and better numerical stability can be obtained at relatively small viscosity. However, we note that the mass is not conserved unless the density is constant (see Section 2). In general, this assumption is not satisfied and such an effect may influence the accuracy of simulation results. Additionally, the shear rate tensor of Yoshino et al. [21] is not computed locally and they adopted a central difference scheme to compute the velocity derivatives, which not only spoils the computational efficiency, but also brings about some difficulties to the implementation of complex boundary conditions with a local lattice scheme.

Taking advantage of the approach in improving numerical stability within the BGK framework, this present work proposes a localized mass-conserving modified lattice Boltzmann model for non-Newtonian fluid flows. Through the Chapman-Enskog analysis, the present model can be demonstrated to recover the macroscopic governing equations. Moreover, as adopted in many previous studies, the shear rate is computed by a local scheme instead of non-local finite-difference schemes.

The outline of this paper is as follows. In Section 2, the lattice Boltzmann equation (LBE) model is proposed for the non-Newtonian fluids together with the Chapman-Enskog analysis. Several numerical examples are carried out to test the model in Section 3, and its superiority over the model of Yoshino et al. [21] is also investigated. This work is concluded in Section 4.