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Reduced Basis Approaches in Time-Dependent Non-Coercive Settings for Modelling the Movement of Nuclear Reactor Control Rods

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Abstract. In this work, two approaches, based on the certified Reduced Basis method, have been developed for simulating the movement of nuclear reactor control rods, in time-dependent non-coercive settings featuring a 3D geometrical framework. In particular, in a first approach, a piece-wise affine transformation based on subdomains division has been implemented for modelling the movement of one control rod. In the second approach, a "staircase" strategy has been adopted for simulating the movement of all the three rods featured by the nuclear reactor chosen as case study. The neutron kinetics has been modelled according to the so-called multi-group neutron diffusion, which, in the present case, is a set of ten coupled parametrized parabolic equations (two energy groups for the neutron flux, and eight for the precursors). Both the reduced order models, developed according to the two approaches, provided a very good accuracy compared with high-fidelity results, assumed as "truth" solutions. At the same time, the computational speed-up in the Online phase, with respect to the fine "truth" finite element discretization, achievable by both the proposed approaches is at least of three orders of magnitude, allowing a real-time simulation of the rod movement and control.

AMS subject classifications: 65M12, 65Y20, 49M25

Key words: Reduced basis method, control rod movement, spatial kinetics, parametrized geometry, multi-group neutron diffusion, non-coercive operators.

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1 Introduction

In the development of the control systems, the preliminary stage of modelling mainly concerns the correct evaluation of the representative system time constants, and getting the fundamental aspects related to the system response to the outside perturbations. In the analysis of the whole nuclear reactor kinetics, which is governed by the neutronics, the most spread approach is constituted by the *point-kinetics* equations [34]. This description of the neutronics is based on a set of coupled non-linear ordinary differential equations that describe both the time-dependency of the neutron population in the reactor and the decay of the delayed neutron precursors, allowing for the main feedback reactivity effects. Among the several assumptions entered in the derivation of these equations, the strongest approximation regards the shape of the neutron flux, which is assumed to be represented by a single, time-independent spatial mode [34].

Nuclear reactors are generally characterised by complex geometries and may feature asymmetric core configurations. Therefore, more accurate and complex modelling approaches might be needed to provide more detailed insights concerning the reactor behaviour during operational transients. It is worth mentioning that innovative reactor concepts, for instance Generation IV reactors [9], feature power density and temperature ranges, experienced by structural materials, such that the corresponding spatial dependency cannot be neglected. Moreover, in order to develop suitable control strategies for such reactors, the spatial effects induced by the movement of the control rods have to be taken into account as well.

In this context, a computational reduced order technique, such as the Reduced Basis (RB) method [27,30], can lead to a simulation tool with real-time simulation, still solving a set of partial differential equations. The goal of a computational reduction technique [23] is to capture the essential features of the input/output behaviour of a system in a rapid, accurate and reliable way, i.e. (i) by improving computational performances and (ii) by keeping the approximation error between the reduced-order solution and the full-order solution under control. In particular, it aims at approximating a parametrized partial differential equation (or a set of partial differential equations) solution with a handful of degrees of freedom instead of thousands or millions that would be needed for a full-order approximation. In this way, the full-order problem has to be solved for a suitable number of instances of the input parameter (through a very demanding Offline computational step, which is performed once), in order to be able to perform many low-cost real-time simulations (inexpensive Online computational step) for several new instances of the parameter.

In the present work, the Reduced Basis method (built upon a high-fidelity "truth" Finite Element (FE) approximation, relying on the `libMesh` library [15]) has been applied to model real-time control rod movement within a nuclear reactor, based on the neutron diffusion coupled equations, simulating a 3D framework, with reference to the TRIGA Mark II nuclear reactor [7] of the University of Pavia (Italy). In particular, two different parametrized models have been considered: a first one, with just one rod, then a second