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Difference Approximation of Stochastic Elastic Equation Driven by Infinite Dimensional Noise

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Abstract. An explicit difference scheme is described, analyzed and tested for numerically approximating stochastic elastic equation driven by infinite dimensional noise. The noise processes are approximated by piecewise constant random processes and the integral formula of the stochastic elastic equation is approximated by a truncated series. Error analysis of the numerical method yields estimate of convergence rate. The rate of convergence is demonstrated with numerical experiments.

AMS subject classifications: 60H15; 65M06

Key words: Stochastic partial differential equations, difference scheme, stochastic elastic equation, infinite dimensional noise, rate of convergence.

1. Introduction

The subject of stochastic partial differential equations (SPDEs) has gained considerable popularity and importance due to its frequent appearance in various fields, such as mechanics, biology, chemistry, epidemiology, microelectronics, economics and finance. SPDEs can describe many phenomena in various fields of science and engineering. During the past decades, there has been an increasing demand for tools and methods of SPDEs in various disciplines and many theoretical analyses for SPDEs have been studied theoretically, for example [9,12,18,24,25,28,30]. The numerical analysis of SPDEs is a young topic of research. Recently, many useful numerical methods for SPDEs have been developed, for instance, finite differences [1–3,6–14,16,19,23,26,27,29,32,33], finite elements [4,5,21,31].

For the contributions on numerical approximating parabolic SPDEs, we refer the reader to [1,2,7,8,10,13,14,16,19,26,29,32,33] and reference therein. In [1] the finite element and difference methods were studied for some linear SPDEs. I. Gyöngy and D. Nualart [13] introduced an implicit numerical scheme for a stochastic parabolic equation and showed that it converges uniformly in probability. I. Gyöngy [14] also

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applied finite difference to stochastic parabolic equations and derived the rate of convergence in L^p. In [15], a finite difference approximation scheme for an elliptic SPDE in dimension $d(d = 1, 2, 3)$ was studied and estimates for the rate of convergence of the approximations were obtained. In [8], the authors studied finite element approximations of some linear parabolic and elliptic SPDEs driven by special additive noises. The effects of the noise on the accuracy of the approximation were discussed. Annie Millet and Pierre-Luc Morine [22] studied the speed of convergence of the explicit and implicit space-time discretization schemes of the solution $u(t, x)$ to a parabolic partial differential equation in any dimension perturbed by a space-correlated Gaussian noise. The influence of the correlation on the speed was observed. For the numerical approximating of hyperbolic SPDEs, we refer the reader to [23, 27] and the reference therein.

It should be noted that most of the papers on numerically approximating SPDEs by finite difference approximation are devoted to the case of space-time white noise. However, there are few papers deal with the SPDEs driven by infinite dimensional noise by a finite difference method.

Enlightened by the above contributions, in this paper we consider strong approximations for a stochastic elastic equation in spatial dimension $d = 1, 2,$ or3 by an explicit difference scheme. To our best knowledge, this is a first step towards the analysis of lattice approximations for stochastic elastic equation driven by infinite dimensional noise.

Let $D = [0, 1]^d, (d = 1, 2, \text{ or } 3)$, consider the numerical approximation of the following stochastic partial differential equation

$$\begin{cases} u_{tt} + \Delta^2 u = f(t, x, u) + \dot{W}(t), \ t \geq 0, x \in D, \\ u = \Delta u = 0 \text{ on } \partial D, \\ u|_{t=0} = u_0, \ u_t|_{t=0} = v_0, \text{ on } D, \end{cases} \tag{1.1}$$

where $W(t)$ is a Hilbert space $U = L^2(D)$ valued Q-Wiener process defined as follows. Q is a symmetric bounded nonnegative operator on $L^2(D)$, there exists a complete orthonormal system $\{e_k\}$ in $L^2(D)$ and a bounded sequence of nonnegative real numbers λ_k such that $Qe_k = \lambda_k e_k, k = 1, 2, \ldots$, then W has the expansion

$$W(t) = \sum_{k=1}^{\infty} \sqrt{\lambda_k} \beta_k(t) e_k. \tag{1.2}$$

And so

$$\dot{W}(t) = \sum_{k=1}^{\infty} \sqrt{\lambda_k} \dot{\beta}_k(t) e_k, \tag{1.3}$$

where $\beta_k(t), (k = 1, 2, \ldots)$ are real valued Brownian notions mutually independent on a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and $\dot{\beta}_k(t)$ is the derivative of $\beta_k(t)$. If $TrQ = \Sigma_{k=1}^{\infty} \lambda_k < +\infty$, then the series (1.2) is convergent in $L^2(\Omega, \mathcal{F}, \mathbb{P}; U)$ and $\dot{W}(t)$ is called colored noise. If $TrQ = +\infty$, then the series (1.2) is not convergent in U, but convergent in a suitable Hilbert space U_1 such that U is embedded continuously into U_1 and