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Application of the Level-Set Model with Constraints in Image Segmentation

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Abstract. We propose and analyze a constrained level-set method for semi-automatic image segmentation. Our level-set model with constraints on the level-set function enables us to specify which parts of the image lie inside respectively outside the segmented objects. Such a-priori information can be expressed in terms of upper and lower constraints prescribed for the level-set function. Constraints have the same conceptual meaning as initial seeds of the popular graph-cuts based methods for image segmentation. A numerical approximation scheme is based on the complementary-finite volumes method combined with the Projected successive over-relaxation method adopted for solving constrained linear complementarity problems. The advantage of the constrained level-set method is demonstrated on several artificial images as well as on cardiac MRI data.

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1. Introduction

The level-set methods for the image segmentation have been studied and applied during the last two decades. The level-set method applied in the image segmentation is typically an iterative method. The segmentation starts with an initial curve $\mathcal{G}^0$ representing an initial guess for the segmented object and it is evolved in the normal direction towards the segmented object by means of a suitable geometric law taking into account the orientation of the segmented object and also the curvature of evolved curves. Loosely speaking, the better the initial guess is, the better and faster the segmentation process is. This is profitable for processing of time sequences where the

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final segmentation of one frame may serve as the initial guess for the next frame. We refer the reader to a wide range of literature on this topic e.g. Caselles *et al.* [7], Handlovičová *et al.* [15], Osher, Paragios [25] or Sethian [28] and references therein. In comparison to parametric models studied by Beneš *et al.* [1] and Kass *et al.* [18] the level-set methods can handle topological changes and therefore one initial curve can split and segment more separate objects. The level-set method is still subject of very active research. In [2], time sequences of 2D MRI slices are segmented as 3D data by the level-set method. It ensures smooth segmentation of adjacent slices. The multi-layer segmentation level-set method for segmentation of images with nested structures is presented in [10]. Combination of the level-set methods with statistical approaches is subject of the review paper [11]. Review of deformable contour models in medical image segmentation can be found in [16].

Among different segmentation methods there are *the graph-cuts methods* (see e.g. Boykov *et al.* [3, 5], Gurholt and Tai [14], Loucký and Oberhuber [20]) which are based on the graph theory and algorithms for finding *minimal cuts* and *the maximal flow* respectively. These algorithms are not iterative and they do not require initial curves. Instead of it, they need initial seeds - one or more points or lines in the interior and exterior of the segmented object.

Each segmentation algorithm requires some description of the object of our interest. The object is described usually in some of the following ways: *Edges* – it is often used information since many objects in the real world have clearly visible edges. In the level-set methods, the Perrona-Malik function serves as an edge detector. *Color or texture pattern* – real objects usually have uniform or homogeneous surface. Therefore areas of the same color or texture pattern belongs very likely to objects of the same type. *Shape* – another criterion might be segmentation of objects with prescribed shape. The object shape can be given by insertions of an appropriate anisotropy [24], the shape-learning methods or by minimizing the elastic energy of the segmentation curve [12]. *Location* – it is expressed by the initial condition. Proper setting of the initial curve for the level-set segmentation may help to specify what object we aim to segment especially if there are more similar objects. Note however, that the initial curve of the level-set method is only an initial step for the segmentation algorithm and the final segmentation may differ from the initial curve significantly. *Skeleton* – the initial seeds in the graph-cuts method differ from the initial curve in one important fact. What is marked by the initial seed as an interior of the segmented object will remain interior even in the final segmentation and vice versa for the exterior.

From this point of view, we can understand the initial seeds in the graph-cuts method as a *hard segmentation constraint* while the initial curve in the level-set method as a *soft segmentation constraint*. In this article, we show how to incorporate local a-priori information similar to the initial seeds used in the graph-cuts method to the level-set method. We propose a new constrained level-set method which can be applied to the image segmentation problems. For better understanding of our method, we will compare it with the classical level-set methods (c.f. [7]) with no surface terms extracting the information about the object color or texture.