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A Markov-Driven Portfolio Execution Strategy with Market Impact

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Dedicated to Xiaoqing Jin on the occasion of his 60th birthday

Abstract. In this paper, we propose a framework for studying optimal agency execution strategies in a Limit Order Book (LOB) under a Markov-modulated market environment. The Almgren-Chriss's market impact model [1] is extended to a more general situation where multiple venues are available for investors to submit trades. Under the assumption of risk-neutrality, a compact recursive formula is derived, using the value iterative method, to calculate the optimal agency execution strategy. The original optimal control problem is then converted to a constrained quadratic optimization problem, which can be solved by using the Quadratic Programming (QP) approach. Numerical examples are given to illustrate the efficiency and effective of our proposed methods.

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Key words: Hamilton-Jacobi-Bellman (HJB) equation, Limit Order Book (LOB), Dynamic Programming (DP) principle, market impact, Quadratic Programming (QP), regime-switching, value iteration method.

1. Introduction

A fundamental part of agency algorithmic trading in equities and other asset classes is trade scheduling. Given a trade target, that is, a number of shares that must be bought or sold in a given time horizon, trade scheduling means a strategic plan for the number of shares to be bought or sold in the horizon. The objective is to optimize some measure of execution quality. For a risk neutral trader, execution quality is measured as the expected cost or income.

Liquidity is an important concern in optimal trade scheduling problem, see, for example, [1–3, 6, 7, 11, 15]. Stocks with low liquidity can have wide bid-ask spreads (see, for

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example [10]). The bid-ask spread, which is the difference between the buying price and the selling price, is incurred as a cost of trading a security. The larger the bid-ask spread, the higher the cost of trading will be. In addition, liquidity also has implications for the price impact of trade. Small orders usually have little impact, especially for liquid stocks. For example, an order to buy 100 shares of a $450 stock with a spread of 1 cent may have no effect on the price. However, an order to buy 100,000 shares may have a significant impact on the price because the volume may be greater than the volume available in the Limit Order Book (LOB) at the best posted quote.

Almgren and Chriss [1] developed an optimal agency execution schedule with the tradeoff between market impact cost and market volatility. The former pushes towards slow trading for reducing the expected execution cost while the latter pushes towards rapid completions of orders for reducing the variation of execution costs. Cartea and Jaimungal [8] proposed an optimal executed policy using both limit and market orders. They obtained the optimal policy by considering different restrictions on volume of both types of orders and depth at which limit orders are posted. They showed how their execution policies performed when targeting the volume schedule of the Almgren-Chriss [1] execution strategy, and proved that their execution strategies outperformed the Almgren-Chriss [1] price with an average savings per share of about one to two basis points.

Due to the tractable and theoretical appeal, most of the studies on optimal execution schedule are based on the premises that the underlying economic environment remains unchanged over time and that there is always sufficient liquidity for trade executions. These assumptions do not seem to reflect well market situations in practice. First, market liquidity varies over time and the market for special assets may become less liquid. Second, as the size of a position increase, the cost and uncertainty associated with liquidating it will increase. In some extreme cases, there may be no price above zero at which the seller can sell the asset.

In this paper, we put forward a framework for studying the optimal liquidation problem in a LOB. Market condition is modeled by a discrete-time finite-state Markov chain, and market liquidity is quantified as the daily liquidation constrain. Following Almgren and Chriss [1], we decompose price impact into parts: temporary price impact and permanent price impact. Temporary impact refers to temporary imbalance in supply/demand caused by our trading. It disappears immediately when trading activity ceases. Permanent impact means changes in the "equilibrium" price due to our trading, which remains at least for the life of our liquidation. We generalize Almgren-Chriss's market impact model [1] to a more general case where multiple venues are available for an investor to submit his/her trades. Based on the principle of arbitrage, an accurate "equilibrium" price formula is derived.

We discuss the optimal agency execution problem with limit and market orders [1] when the trader employs the following strategy: (i) submits limit sell orders at the best ask quotes and is committed to sell *one* share of stock at these prices, should he/she meets counterpart market orders. Otherwise (ii) posts market orders for an immediate execution.

[1]When a trader places an order to sell or buy a stock, there are two types of execution options: market order and limit order. For a market order, the instruction is to execute as quickly as possible at the present market price. While for a limit order, the instruction is to execute at a given bid or ask price.