Communications
in
Mathematical
Research
33(4)(2017), 304–310

Existence of Solutions to a p-Laplacian Equation with Integral Initial Condition

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Communicated by Gao Wen-jie

Abstract: In this paper, a class of one-dimension p-Laplacian equation with nonlocal initial value is studied. The existence of solutions to such a problem is obtained by using the topological degree method.

Key words: integral initial condition, existence, topological

2010 MR subject classification: 34B15, 37C25

Document code: A

Article ID: 1674-5647(2017)04-0304-07

DOI: 10.13447/j.1674-5647.2017.04.02

1 Introduction

Boundary value problems with integral boundary conditions in the ordinary differential equations arise in different areas of applied mathematics and physics. For example, heat conduction, chemical engineering, underground water flow, thermo-elasticity and plasma physics can be reduced to nonlocal problems with integral boundary conditions. For boundary-value problems with integral boundary conditions and comments on their importance, we refer the readers to [1]–[4] and the references therein. For more information about the general theory of integral equations and their relation with boundary-value problems, readers may refer to [5]–[7].

In recent years, the existence and multiplicity of positive solutions for nonlocal problems have attracted great attention to many mathematicians. Readers may refer to [8]–[15] and references therein. On the other hand, initial-value problems with integral conditions constitute a very interesting and important class of problems. The integral initial value

Received date: May 30, 2016.

Foundation item: The NSF (11271154 and 11326103) of China, Research Project (2014164) of the Education of Jilin Province, and the Youth Studies Program (XJ2012006) of Jilin University of Finance and Economics.

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problems to one-dimension p-Laplacian equation are meaningful in mathematics and physics.

The purpose of this paper is to investigate the existence of solutions to the following one-dimension p-Laplacian equation:

$$(\phi(u'))' = -q(t)f(t,u,u'), \qquad t \in (0,1) \tag{1.1}$$

with integral initial value

$$u(0) = \int_0^1 g(s)u(s)\mathrm{d}s, \tag{1.2}$$

$$u'(0) = A, \tag{1.3}$$

where

$$\phi(s) = \phi_p(s) = |s|^{p-2}s,$$
$$\phi^{-1}(s) = \phi_q(s) = |s|^{q-2}s,$$

$p, q > 1$, $\dfrac{1}{p} + \dfrac{1}{q} = 1$, $f : [0,\, 1] \times \mathbf{R} \times \mathbf{R} \to \mathbf{R}$ is continuous, $q(t),\ g(s) \in L^1([0,\, 1])$ and A is a real constant. Our proof is based upon Leray-Schauder topological degree.

The paper is organized as follows. In Section 2, we provide some necessary preliminaries and in Section 3 the main result is stated and proved.

2 Preliminaries

Let I be the real interval $[0,\, 1]$ and $C(I)$ be the Banach space of all continuous functions $u : I \to \mathbf{R}$, equipped with the norm

$$\|u\|_0 = \max\{|u(t)|; t \in I\}$$

for any $u \in C(I)$.

Consider the following problem:

$$(\phi(x'))' = -y(t), \qquad t \in (0,1), \tag{2.1}$$

$$x(0) = \int_0^1 g(s)x(s)\mathrm{d}s, \tag{2.2}$$

$$x'(0) = A, \tag{2.3}$$

where

$$y(t) \in C(I), \qquad \int_0^1 g(s)\mathrm{d}s \neq 1.$$

Integrating (2.1) from 0 to t, we obtain

$$\phi(x'(t)) - \phi(x'(0)) = -\int_0^t y(s)\mathrm{d}s,$$

and by using the initial condition (2.3), we have

$$x'(t) = \phi^{-1}\left(\phi(A) - \int_0^t y(s)\mathrm{d}s\right).$$

Integrating the above equality from 0 to t again, we obtain

$$x(t) - \int_0^1 g(s)x(s)\mathrm{d}s = \int_0^t \phi^{-1}\left(\phi(A) - \int_0^\tau y(s)\mathrm{d}s\right)\mathrm{d}\tau. \tag{2.4}$$