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Shared Values of Certain Nonlinear Differential Polynomials of Meromorphic Functions

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Abstract: In this paper, by using the idea of truncated counting functions of meromorphic functions, we deal with the problem of uniqueness of the meromorphic functions whose certain nonlinear differential polynomials share one finite nonzero value.

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1 Introduction and Main Results

In this paper, by meromorphic function we always means a function which is meromorphic in the whole complex plane **C**. We adopt the usual and standard notations in the Nevanlinna theory of meromorphic functions such as $T(r, f)$, $N(r, f)$, $\bar{N}(r, f)$, $m(r, f)$ and so on (see [1]–[3]). For a meromorphic function f, we denote by $S(r, f)$ any quantity satisfying $S(r, f) = o\{T(r, f)\}$, as $r \to +\infty$, possibly outside a set of finite measure.

Let f be a nonconstant meromorphic function, a be a complex number, and m be a positive integer. We denote by $E(a, f)$ the set of zeros of $f-a$, where a zero with multiplicity m is counted m times in the set. If these zeros are only counted once, then we denote the set by $\bar{E}(a, f)$. In addition, we denote by $E_{m)}(a, f)$ the set of zeros of $f-a$ with multiplicity $l \le m$, where a zero with multiplicity $l \le m$ is counted l times in the set. Similarly, if these zeros are only counted once, then we denote the set by $\bar{E}_{m)}(a, f)$.

Let f and g be two nonconstant meromorphic functions, and a be a complex number. If $f-a$ and $g-a$ have the same zeros with the same multiplicities, i.e., $E(a, f) = E(a, g)$,

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then we say that f and g share a CM (counting multiplicities). And if we do not consider the multiplicities, i.e., $\bar{E}(a, f) = \bar{E}(a, g)$, then we say that f and g share a IM (ignoring multiplicities).

For any constant $a \in \mathbf{C} \cup \{\infty\}$, we define

$$\delta(a, f) = 1 - \limsup_{r\to\infty} \frac{N\left(r, \dfrac{1}{f-a}\right)}{T(r, f)},$$

$$\Theta(a, f) = 1 - \limsup_{r\to\infty} \frac{\bar{N}\left(r, \dfrac{1}{f-a}\right)}{T(r, f)}.$$

Also we use the notations $\delta(a)$ and $\Theta(a)$, where $\delta(a) = \min\{\delta(a, f),\ \delta(a, g)\}$ and $\Theta(a) = \min\{\Theta(a, f),\ \Theta(a, g)\}$.

Meanwhile, the order λ and the lower order μ of a meromorphic function f are defined in turn as follows:

$$\lambda := \lambda(f) = \limsup_{r\to\infty} \frac{\log T(r, f)}{\log r},$$

$$\mu := \mu(f) = \liminf_{r\to\infty} \frac{\log T(r, f)}{\log r}.$$

Nevanlinna[4] pointed out that if f and g share four distinct values CM, then f is a Möbius transformation of g. In recent decades, related to one famous conjecture of Hayman[5], many interesting uniqueness theorems for certain types of differential polynomials with one shared value were obtained.

In 2002, Fang[6] proved the following uniqueness theorem.

Theorem 1.1 *Let f and g be two nonconstant entire functions, and n, k be two positive integers with $n \geq 2k+8$. If $(f^n(f-1))^{(k)}$ and $(g^n(g-1))^{(k)}$ share* 1 CM, *then $f \equiv g$.*

In 2008, Chen *et al.*[7] generalized Theorem 1.1 by deriving the following result.

Theorem 1.2 *Let f and g be two nonconstant entire functions, and let n, k be two positive integers with $n > 5k+13$. If $(f^n(f-1))^{(k)}$ and $(g^n(g-1))^{(k)}$ share 1 IM, then $f \equiv g$.*

In 2011, Lin X. Q. and Lin W. C.[8] proved the following result which was an improvement of Theorem 1.2.

Theorem 1.3 *Let f and g be two nonconstant entire functions, and n, k and $m \geq 2$ be three positive integers with $n > k+(4k+7)(1-\Theta(0))+4(1-\delta(1))$. If $\bar{E}_{m)}(1, (f^n(f-1))^{(k)}) = \bar{E}_{m)}(1, (g^n(g-1))^{(k)})$, then $f \equiv g$.*

The above results have undergone various extensions from different directions. In 2014, by using Zalcman's lemma, Li and Yi[9] considered the case of meromorphic functions and proved the following result which was a generalization of Theorem 1.2.