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L^2-harmonic 1-forms on Complete Manifolds

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Abstract: We study the global behavior of complete minimal δ-stable hypersurfaces in $\mathbf{R}^{n+1}$ by using L^2-harmonic 1-forms. We show that a complete minimal δ-stable $\left(\delta > \dfrac{(n-1)^2}{n^2}\right)$ hypersurface in $\mathbf{R}^{n+1}$ has only one end. We also obtain two vanishing theorems of complete noncompact quaternionic manifolds satisfying the weighted Poincaré inequality. These results are improvements of the first author's theorems on hypersurfaces and quaternionic Kähler manifolds.

Key words: minimal hypersurface, end, quaternionic manifold, weighted Poincaré inequality

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1 Introduction

Palmer[1] showed that there is no non-trivial L^2-harmonic 1-form on a complete stable minimal hypersurface in $\mathbf{R}^{n+1}$. Cao *et al.*[2] proved that a complete stable minimal hypersurface in $\mathbf{R}^{n+1}$ $(n \geq 3)$ must have only one end. Cheng *et al.*[3] showed that a complete oriented weakly stable minimal hypersurface in $\mathbf{R}^{n+1}$ $(n \geq 3)$ must contain no nonconstant bounded harmonic functions with finite Dirichlet integral and have only one end. If the ambient manifold is not the Euclidean space, Cheng[4] gave one end theorem for complete noncompact oriented stable minimal hypersurfaces immersed in an $(n+1)$-dimensional $(n \geq 3)$ complete oriented manifold of positive sectional curvature. Recently, by use of the rigidity of complete Riemannian manifolds with weighted Poincaré inequality, Cheng and Zhou[5] showed that: if M is an $\dfrac{n-2}{n}$-stable complete minimal hypersurface in $\mathbf{R}^{n+1}$ $(n \geq 3)$ and it has bounded

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norm of second fundamental form, then M either has only one end or is a catenoid. The first author proved that if M^n $(n \geq 2)$ is a complete minimal δ-stable $\left(\delta > \frac{(n-1)^2}{n^2}\right)$ hypersurface in $\mathbf{R}^{n+1}$ and it has the bounded norm of the second fundamental form, then the space of L^2 integrable harmonic 1-forms $H^1(L^2(M))$ is trivial (see [6], Corollary 2.5).

In this paper, firstly, we can obtain the following result:

Theorem 1.1 *Suppose that M^n $(n \geq 2)$ is a complete minimal δ-stable $\left(\delta > \frac{(n-1)^2}{n^2}\right)$ hypersurface in $\mathbf{R}^{n+1}$. Then the space of L^2 integrable harmonic 1-forms $H^1(L^2(M))$ is trivial and M has only one end.*

Remark 1.1 Theorem 1.1 generalizes Corollary 2.5 in [6] without the restriction of the second fundamental forms.

Secondly, Lam[7] showed that if M^{4n} is a $4n$-dimensional complete noncompact quaternionic Kähler and the Ricci curvature of M satisfies

$$\mathrm{Ric}_M \geq -\frac{4}{3}\lambda_1(M) + \delta$$

for a positive constant δ, where $\lambda_1(M)$ is the lower bound of the spectrum of the Laplacian on M, then

$$H^1(L^2(M)) = \{0\}.$$

Suppose that M is a $4n$-dimensional complete noncompact quaternionic manifold satisfying the weighted Poincaré inequality with a non-negative weight function $\rho(x)$ and the Ricci curvature satisfies

$$\mathrm{Ric}_M(x) \geq -\frac{4}{3}\rho(x) + \sigma(x)$$

for a nonnegative continuous function σ $(\sigma \neq 0)$. If $\rho(x) = O(r_p^{2-\alpha})$, where $r_p(x)$ is the distance function from x to some fixed point p and $0 < \alpha < 2$, then $H^1(L^2(M)) = \{0\}$ (see [6]). It is interesting to see if a similar theorem holds without the restriction of growth rate of the weight function. The following theorems had been established:

Theorem 1.2 *Suppose that M is a $4n$-dimensional complete noncompact quaternionic manifold satisfying the weighted Poincaré inequality with a non-negative continuous weight function $\rho(x)$ ($\rho(x)$ is not identically zero). Assume that the Ricci curvature satisfies*

$$\mathrm{Ric}_M(x) \geq -\alpha\rho(x)$$

for a constant α with $0 < \alpha < \frac{4}{3}$. Then $H^1(L^2(M)) = \{0\}$.

Theorem 1.3 *Suppose that M is a $4n$-dimensional complete noncompact quaternionic manifold satisfying the weighted Poincaré inequality with a non-negative continuous weight function $\rho(x)$. Assume that the Ricci curvature satisfies*

$$\mathrm{Ric}_M(x) \geq -\alpha\rho(x) - \beta$$

for constants α with $0 < \alpha < \frac{4}{3}$ and $\beta > 0$. If the lower bound of the spectrum $\lambda_1(M)$ of the