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# Long-time Dynamics for Thermoelastic Bresse System of Type III

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**Abstract:** This paper considers the thermoelastic beam system of type III with friction dissipations acting on the whole system. By using the methods developed by Chueshov and Lasiecka, we get the quasi-stability property of the system and obtain the existence of a global attractor with finite fractal dimension. Result on exponential attractors of the system is also proved.

**Key words:** Bresse system, quasi-stability, global attractor, fractal dimension, exponential attractor

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## 1 Introduction

In this paper, we consider a semilinear thermoelastic Bresse system of Type III

$$\begin{cases} \rho_1\varphi_{tt} - k(\varphi_x + \psi + l\omega)_x - k_0 l(\omega_x - l\varphi) + g_1(\varphi_t) + f_1(\varphi, \psi, \omega) = h_1, \\ \rho_2\psi_{tt} - b\psi_{xx} + k(\varphi_x + \psi + l\omega) + \delta\theta_x + g_2(\psi_t) + f_2(\varphi, \psi, \omega) = h_2, \\ \rho_1\omega_{tt} - k_0(\omega_x - l\varphi)_x + kl(\varphi_x + \psi + l\omega) + g_3(\omega_t) + f_3(\varphi, \psi, \omega) = h_3, \\ \rho_3\theta_{tt} - k_1\theta_{xx} + \delta\psi_{ttx} + \displaystyle\int_0^\infty \xi(s)\theta_{xx}(t-s)\mathrm{d}s + f_4(\theta) = h_4, \end{cases} \tag{1.1}$$

for $x \in (0, L)$ and $t > 0$, with the initial conditions

$$\begin{cases} (\varphi(0), \psi(0), \omega(0), \theta(0)) = (\varphi_0, \psi_0, \omega_0, \theta_0), \\ (\varphi_t(0), \psi_t(0), \omega_t(0), \theta_t(0)) = (\varphi_1, \psi_1, \omega_1, \theta_1), \\ \theta(-s)|_{s>0} = \vartheta_0(s), \end{cases} \tag{1.2}$$

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where the past history function $\vartheta_0$ on $\mathbf{R}^+ = (0, +\infty)$ is a given datum, and the boundary conditions are as follows:

$$\begin{cases} \varphi(0, t) = \psi(0, t) = \omega(0, t) = \theta(0, t) = 0, \\ \varphi(L, t) = \psi(L, t) = \omega(L, t) = \theta(L, t) = 0, \end{cases} \tag{1.3}$$

where $\varphi$, $\psi$, $\omega$, $\theta$ represent, respectively, vertical displacement, shear angle, longitudin displacement and relative temperature. The coefficients $\rho_1$, $\rho_2$, $\rho_3$, $b$, $k$, $k_0$, $k_1$, $\delta$ are positive constants related to the material and the parameter $l$ stands for the curvature of the beam. The function $g_1$, $g_2$, $g_3$ are nonlinear damping terms, the functions $f_1$, $f_2$, $f_3$, $f_4$ are nonlinear source terms, and $h_1$, $h_2$, $h_3$, $h_4$ are external force terms. Elastic structures of the arches type are object of study in many areas like mathematics, physics and engineering. For more details, the interested reader can visit the works of Liu and Rao[1], Boussouira *et al.*[2] and reference therein.

Throughout the paper, the assumptions are always made as follows.

• $f_1$, $f_2$, $f_3$, $f_4$ are nonlinear source terms. Assume that there exists a non-negative $C^2$ function $F : \mathbf{R}^3 \to \mathbf{R}$ such that

$$\nabla \boldsymbol{F} = (f_1,\, f_2,\, f_3), \tag{1.4}$$

and there exists a constant $C_f > 0$ such that

$$|\nabla f_i(u,\, v,\, w)| \le C_f(1 + |u|^{p-1} + |v|^{p-1} + |w|^{p-1}), \qquad i = 1, 2, 3,\ p \ge 1. \tag{1.5}$$

Furthermore, assume that $F$ is homogeneous of order $p + 1$,

$$F(\lambda(u,\, v,\, w)) = \lambda^{p+1} F(u,\, v,\, w), \qquad \lambda > 0, \quad (u,\, v,\, w) \in \mathbf{R}^3.$$

Since $F$ is homogeneous, the Euler homogeneous function theorem yields the following useful identity:

$$\begin{aligned} f_1(u,\, v,\, w)u + f_2(u,\, v,\, w)v + f_3(u,\, v,\, w)w &= \nabla \boldsymbol{F}(u,\, v,\, w)(u,\, v,\, w) \\ &= (p + 1)F(u,\, v,\, \omega). \end{aligned} \tag{1.6}$$

By (1.5), we derive that there exists a constant $C_F > 0$ such that

$$F(u,\, v,\, w) \le C_F(1 + |u|^{p+1} + |v|^{p+1} + |w|^{p+1}). \tag{1.7}$$

Assume that there exists a non-negative $C^2$ function $\bar{F} : \mathbf{R} \to \mathbf{R}$ such that

$$\bar{F}(\theta) = \int_0^\theta f_4(s)\mathrm{d}s, \tag{1.8}$$

and there exists a positive constant $\bar{C}_f$ such that

$$f_4(0) = 0, \quad |f_4'(s)| \le \bar{C}_f(1 + |s|^{p-1}), \qquad s \in \mathbf{R}, \tag{1.9}$$

with $p \ge 1$.

$$(f_4(s_2) - f_4(s_1))(s_2 - s_1) + \lambda_1(s_2 - s_1)^2 \ge 0, \qquad s_1,\, s_2 \in \mathbf{R}, \tag{1.10}$$

and there exists a constant $\kappa \in (0, 1)$ and $m_f > 0$ such that

$$f_4(s)s + (1 - \kappa)\lambda_1 s^2 + m_f \ge 0, \qquad s \in \mathbf{R}, \tag{1.11}$$

where $\lambda_1 = \dfrac{\pi^2}{L^2}$ is the first positive eigenvalue of $-\Delta$ in $(0, L)$ with zero Dirichlet boundary conditions.

• $g_i (i = 1, 2, 3)$ is damping term satisfying

$$g_i \in C_1(\mathbf{R}), \quad g_i(0) = 0, \quad \text{and} \quad g_i \text{ is increasing}, \tag{1.12}$$