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Hopf Bifurcation of Delayed Predator-prey System with Reserve Area for Prey and in the Presence of Toxicity

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Abstract: A kind of three species delayed predator-prey system with reserve area for prey and in the presence of toxicity is proposed in this paper. Local stability of the coexistence equilibrium of the system and the existence of a Hopf bifurcation is established by choosing the time delay as the bifurcation parameter. Explicit formulas to determine the direction and stability of the Hopf bifurcation are obtained by means of the normal form theory and the center manifold theorem. Finally, we give a numerical example to illustrate the obtained results.

Key words: delay, Hopf bifurcation, predator-prey system, periodic solution, toxicity

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1 Introduction

In recent decades, it has been of great interest to investigate the dynamic interaction between predator species and prey species in both ecology and mathematical ecology (see [1]–[3]). Especially, two species predator-prey systems have been investigated by many scholars at home and abroad (see [4]–[10]), since the pioneering works by Lotka[11] and Volterra[12]. However, two species predator-prey systems can describe only a small number of the phenomena that are commonly observed in nature (see [13]). Therefore, it is more realistic

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to consider a predator-prey system with three or more species to understand complex dynamical behaviors of multiple species predator-prey systems in the real world. Inspired by this idea, Yang and Jia[14] proposed the following three species predator-prey system with reserve area for prey and in the presence of toxicity:

$$\begin{cases} \dfrac{\mathrm{d}x(t)}{\mathrm{d}t} = r_1 x(t)\left(1 - \dfrac{x(t)}{K}\right) - \sigma_1 x(t) + \sigma_2 y(t) - v_1 x^2(t) - \dfrac{a x(t) z(t)}{b + x(t)} - q_1 E x(t), \\ \dfrac{\mathrm{d}y(t)}{\mathrm{d}t} = r_2 y(t) + \sigma_1 x(t) - \sigma_2 y(t) - v_2 y^2(t), \\ \dfrac{\mathrm{d}z(t)}{\mathrm{d}t} = \dfrac{\beta a x(t) z(t)}{b + x(t)} - d z(t) - \varpi z(t) - q_2 E z(t), \end{cases} \tag{1.1}$$

where $x(t)$, $y(t)$ and $z(t)$ denote the biomass densities of the prey species in the unreserve areas, the prey species in the reserve areas and the predator species at time t, respectively. K is the carrying capacity of the prey species in the unreserve areas; r_1 is the intrinsic growth rate of the prey species in the unreserve areas; r_2 is the birth rate of the prey species in the reserve areas; v_1, v_2 and ϖ are the infection rates of the prey species in the unreserve areas, the prey species in the reserve areas and the predator species by an external toxic substance, respectively; q_1 and q_2 are the catchability coefficients of the prey species in the unreserved areas and the predator species, respectively; E is the effort applied to harvest the prey species and the predator species in the unreserve areas; σ_1 (σ_2) is the rate at which the prey species in the unreserve (reserve) areas migrate into the reserve (unreserve) areas; d is the death rate of the predator species; a is the capturing rate of the predator species; β is the rate of conversing the prey species in the unreserve areas into the predator species; b is the half saturation rate of the predator species.

Time delays of one type or another have been incorporated into predator-prey systems by many scholars (see [15]–[21]), since delay differential equations exhibit much more complicated dynamics than ordinary ones. Motivated by the work above and considering that the consumption of the prey species by the predator species throughout its past history governs the present birth rate of the predator species, we incorporate the time delay due to the gestation of the predator species into system (1.1) and get the following predator-prey system with time delay:

$$\begin{cases} \dfrac{\mathrm{d}x(t)}{\mathrm{d}t} = r_1 x(t)\left(1 - \dfrac{x(t)}{K}\right) - \sigma_1 x(t) + \sigma_2 y(t) - v_1 x^2(t) - \dfrac{a x(t) z(t)}{b + x(t)} - q_1 E x(t), \\ \dfrac{\mathrm{d}y(t)}{\mathrm{d}t} = r_2 y(t) + \sigma_1 x(t) - \sigma_2 y(t) - v_2 y^2(t), \\ \dfrac{\mathrm{d}z(t)}{\mathrm{d}t} = \dfrac{\beta a x(t-\tau) z(t-\tau)}{b + x(t-\tau)} - d z(t) - \varpi z(t) - q_2 E z(t), \end{cases} \tag{1.2}$$

where τ is the time delay due to the gestation of the predator species.

The rest of this work is organized in this pattern. In the next section, the existence of the Hopf bifurcation is investigated. In Section 3, based on the normal form method and center manifold theory, properties of the Hopf bifurcation are investigated. In Section 4, a numerical example is carried out in order to support the obtained theoretical predictions. The final section gives our conclusion.