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Fekete-Szegö Inequality for a Subclass of Bi-univalent Functions Associated with Hohlov Operator and Quasi-subordination

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Abstract: In this paper, we introduce a new subclass of bi-univalent functions defined by quasi-subordination and Hohlov operator and obtain the coefficient estimates and Fekete-Szegö inequality for function in this new subclass. The results presented in this paper improve or generalize the recent works of other authors.

Key words: analytic function, univalent function, bi-univalent function, coefficient estimate, Fekete-Szegö inequality, Hohlov operator, quasi-subordination

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1 Introduction

Let H denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \tag{1.1}$$

which are analytic in the open unit disk $U = \{z\colon |z| < 1\}$. Further, by S we denote the family of all functions in H which are univalent in U.

In [1], Robertson introduced the concept of quasi-subordination. For two analytic func-

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tions f and φ, the function f is quasi-subordination to φ written as

$$f(z) \prec_q \varphi(z), \qquad z \in U,$$

if there exist analytic functions h and ω with $|h(z)| \leq 1$, $\omega(0) = 0$ and $|\omega(z)| < 1$ such that

$$\frac{f(z)}{h(z)} \prec \varphi(z),$$

which is equivalent to

$$f(z) = h(z)\varphi(\omega(z)), \qquad z \in U.$$

Observe that if $h(z) \equiv 1$, then the quasi-subordination reduces to be subordination. Also, if $\omega(z) = z$, then

$$f(z) = h(z)\varphi(z),$$

and in this case we say that $f(z)$ is majorized by $\varphi(z)$ and it is written as

$$f(z) \ll \varphi(z), \qquad z \in U.$$

Obviously, the quasi-subordination is the generalization of subordination as well as majorization.

For the functions $f,\, g \in H$, where $f(z)$ is given by (1.1) and

$$g(z) = z + \sum_{n=2}^{\infty} b_n z^n,$$

the Hadamard product or convolution is denoted by $f * g$ and is defined by:

$$(f * g)(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n, \tag{1.2}$$

and the Gaussian hypergeometric function ${}_2F_1(a,\, b,\, c;\, z)$ for the complex parameters a, b and c with $c \neq 0,\, -1,\, -2, -3, \cdots$ is defined by:

$${}_2F_1(a,\, b,\, c;\, z) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n} \frac{z^n}{n!} = 1 + \sum_{n=2}^{\infty} \frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}} \frac{z^{n-1}}{(n-1)!}, \qquad z \in U, \tag{1.3}$$

where $(l)_n$ denotes the Pochhammer symbol (the shifted factorial) defined by:

$$(l)_n = \frac{\Gamma(l+n)}{\Gamma(l)} = \begin{cases} 1, & n = 0,\ l \in \mathbf{C}\backslash\{0\}; \\ l(l+1)\cdots(l+n-1), & n = 1, 2, 3, \cdots. \end{cases}$$

For the positive real values a, b and c ($c \neq 0, -1, -2, -3, \cdots$), Hohlov[2]–[3] introduced a convolution operator $I_{a,b;c}$ by using the Gaussian hypergeometric function ${}_2F_1(a,\, b,\, c;\, z)$ given by (1.3) as follows:

$$I_{a,b;c} f(z) = z\,{}_2F_1(a,\, b,\, c;\, z) * f(z) = z + \sum_{n=2}^{\infty} y_n a_n z^n, \qquad z \in U, \tag{1.4}$$

where

$$y_n = \frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(n-1)!}. \tag{1.5}$$

Observe that, if $b = 1$ in (1.5), then the Hohlov operator $I_{a,b;c}$ reduces to the Carlson-Shaffer operator. Also it can be easily seen that the Hohlov operator is a generalization of the Ruscheweyh derivative operator and the Bernardi-Libera-Livingston operator.

It is well known that every function $f \in S$ has an inverse f^{-1}, which is defined by

$$f^{-1}(f(z)) = z, \qquad z \in U$$