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Transition Operator Approach to Seismic Full-Waveform Inversion in Arbitrary Anisotropic Elastic Media

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Abstract. We generalize the existing distorted Born iterative T-matrix (DBIT) method to seismic full-waveform inversion (FWI) based on the scalar wave equation, so that it can be used for seismic FWI in arbitrary anisotropic elastic media with variable mass densities and elastic stiffness tensors. The elastodynamic wave equation for an arbitrary anisotropic heterogeneous medium is represented by an integral equation of the Lippmann-Schwinger type, with a 9-dimensional wave state (displacement-strain) vector. We solve this higher-dimensional Lippmann-Schwinger equation using a transition operator formalism used in quantum scattering theory. This allows for domain decomposition and novel variational estimates. The tensorial nonlinear inverse scattering problem is solved iteratively by using an expression for the Fréchet derivatives of the scattered wavefield with respect to elastic stiffness tensor fields in terms of modified Green's functions and wave state vectors that are updated after each iteration. Since the generalized DBIT method is consistent with the Gauss-Newton method, it incorporates approximate Hessian information that is essential for the reduction of multi-parameter cross-talk effects. The DBIT method is implemented efficiently using a variant of the Levenberg-Marquard method, with adaptive selection of the regularization parameter after each iteration. In a series of numerical experiments based on synthetic waveform data for transversely isotropic media with vertical symmetry axes, we obtained a very good match between the true and inverted models when using the traditional Voigt parameterization. This suggests that the effects of cross-talk can be sufficiently reduced by the incorporation of Hessian information and the use of suitable regularization methods. Since the generalized DBIT method for FWI in anisotropic elastic media is naturally target-oriented, it may be particularly suitable for applications to seismic reservoir characterization and monitoring. However, the theory and method presented here is general.

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1 Introduction

Seismic full-waveform inversion (FWI) promises images of the underground that are sharper and of higher resolution than those in conventional migration velocity analysis and travel time tomography [1–5]. However, FWI is characterized by several challenges [4], including a very strong sensitivity of the inversion to the initial model and a very high computational cost. The sensitivity of the FWI result to the starting model may be reduced by the use of multi-scale regularization methods [6, 7], the calculation of higher-order Fréchet derivatives [8], envelope inversion [9] or some kind of renormalization [10, 11]. Also, the computational cost may be significantly reduced using a simultaneous source method [12] and/or some kind of domain decomposition [13, 14].

Many FWI methods have been developed on the basis of the acoustic wave equation [1–19]. The acoustic approximation may be adequate for many imaging purposes. For applications of the FWI method to seismic reservoir characterization and monitoring, however, it is essential to use the elastodynamic wave equation [20–25]. This is because shear-waves contain important information about the nature and content of pores and cracks [22, 26, 27]. To increase the realism of FWI, it may also be important to account for the effects of seismic anisotropy [28–44].

When a seismic wave propagates in a heterogeneous medium the wavelength is often large compared to the scale-size of pores, cracks and other heterogeneities so that the wave cannot 'see the details of the microstructure but only a smeared-out or averaged structure. If the heterogeneous microstructure has certain preferred directions then the average response on the macroscopic level will be anisotropic [26]. Many reservoirs are transversely isotropic due to fine layering [45] and/or the presence of aligned fractures [46]. Also, the shale layers that typically surround the reservoir formations are known to be transversely isotropic due to the presence of aligned clay minerals [47, 48]. The Curie-Neumann principle suggests that if the microstructure has certain symmetries then the macroscopic response will show the same symmetries [27, 49]. However, it is important to keep in mind that symmetries in geology are always approximate [49]. In any case, seismic anisotropy is an academically interesting topic of great practical interest.

The goal of FWI in anisotropic elastic media is to reconstruct multiple parameter fields from the observed waveforms. In a general medium, there are 21 independent elastic stiffness parameters and the mass density [27]. The number of independent parameters can, of course, be reduced when material symmetries are present [27], but multi-parameter FWI can be very challenging even for just a few independent parameters [36]. Involving multiple parameters increases the nonlinearity of the inversion process and also introduces parameter crosstalk; that is, the influence of one elastic parameter on the data associated with another elastic parameter [32, 39, 50].