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Existence of Solutions for a Parabolic System Modelling Chemotaxis with Memory Term

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Abstract. In this paper, we come up with a parabolic system modelling chemotaxis with memory term, and establish the local existence and uniqueness of weak solutions. The main methods we use are the fixed point theorem and semigroup theory.

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1 Introduction

We consider in this paper a parabolic-parabolic system modelling chemotaxis, which explains the movement of population density or the movement of single particles. The movement behavior of many species is guided by external signals, for instance, amoebas move upwards chemical gradients, insects orient towards light sources, the smell of a sexual partner makes it favorable to choose a certain direction.

The basic model of chemotaxis was introduced by Keller and Segel in [1], and this reads

$$u_t = \nabla(\nabla u - \chi(v)u\nabla v), \tag{1.1}$$

$$\varepsilon v_t = \Delta v + g(u,v), \tag{1.2}$$

where u represents the population density and v denotes the density of the external stimulus, χ is the sensitive coefficient, the time constant $0 \leq \varepsilon \leq 1$ indicates that the spatial spread of the organisms u and the signal v are on different time scales. The case $\varepsilon = 0$ corresponds to a quasi-steady state assumption for the signal distribution.

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Since the KS model is designed to describe the behavior of bacteria, the question arises whether or not this model is able to show aggregation. A number of theoretical research found exact conditions for aggregations and for blow-up [2–16]. Besides, free boundary problems for the chemotaxis model is considered [17–24].

The possibility of blow-up has been shown to depend strongly on space dimension. For instance, in the case of χ is a constant, and $g(u,v)=-\gamma v+\alpha u$, finite time blow-up never occurs in 1-D case [25] (unless there is no diffusion of the attractant v, [26, 27]), but can always occur in N-D cases for $N\geq 3$. For the 2-D case, it depends on the initial data, i.e. there exists a threshold; if the initial distribution exceeds its threshold, then the solution blows up in finite time, otherwise the solution exists globally [28].

We introduce a memory term in equation (1.1), i.e.

$$u_t=\nabla(\nabla u-\chi(v)u\nabla v)+u^q\int_0^t u^p(\cdot,\tau)\mathrm{d}\tau, \tag{1.3}$$

where $p,q\geq 0$ are nonnegative constants.

There are two main sources of motivation for our investigation of (1.3). One of them is the semilinear heat equation model with memory,

$$\begin{cases} u_t=\Delta u+u^q\int_0^t u^p(\cdot,\tau)\mathrm{d}\tau & \text{in } \Omega\times(0,T),\\ u(\cdot,0)=u_0 & \text{in } \Omega\times\{0\},\\ u=0 & \text{on } \partial\Omega\times(0,T), \end{cases} \tag{1.4}$$

and the other is

$$\begin{cases} u_t=\Delta u & \text{in } \Omega\times(0,T),\\ u(\cdot,0)=u_0 & \text{in } \Omega\times\{0\},\\ \dfrac{\partial u}{\partial n}=u^q\int_0^t u^p(\cdot,\tau)\mathrm{d}\tau & \text{on } \partial\Omega\times(0,T). \end{cases} \tag{1.5}$$

It is known that the solutions of (1.4) and (1.5) are global in the case of $0\leq p+q\leq 1$ [29,30].

Let $g(u,v)=-\gamma v+\alpha u$, $\chi(v)=\chi$ is a constant, and supplement equations (1.3), (1.2) with some initial boundary conditions, then we obtain the following system

$$\begin{cases} u_t=\nabla(\nabla u-\chi u\nabla v)+u^q\int_0^t u^p(\cdot,\tau)\mathrm{d}\tau & \text{in } \Omega\times(0,T),\\ \varepsilon v_t=\Delta v-\gamma v+\alpha u & \text{in } \Omega\times(0,T),\\ u(\cdot,0)=u_0 & \text{in } \Omega\times\{0\},\\ v(\cdot,0)=v_0 & \text{in } \Omega\times\{0\},\\ \dfrac{\partial u}{\partial n}=\dfrac{\partial v}{\partial n}=0 & \text{on } \partial\Omega\times(0,T). \end{cases} \tag{1.6}$$