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COUPLED NONLOCAL NONLINEAR SCHRÖDINGER EQUATION AND N-SOLITON SOLUTION FORMULA WITH DARBOUX TRANSFORMATION*†

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Abstract

Ablowitz and Musslimani proposed some new nonlocal nonlinear integrable equations including the nonlocal integrable nonlinear Schrödinger equation. In this paper, we investigate the Darboux transformation of coupled nonlocal nonlinear Schrödinger (CNNLS) equation with a spectral problem. Starting from a special Lax pairs, the CNNLS equation is constructed. Then, we obtain the one-, two- and N-soliton solution formulas of the CNNLS equation with N-fold Darboux transformation. Based on the obtained solutions, the propagation and interaction structures of these multi-solitons are shown, the evolution structures of the one-dark and one-bright solitons are exhibited with $N = 1$, and the overtaking elastic interactions among the two-dark and two-bright solitons are considered with $N = 2$. The obtained results are different from those of the solutions of the local nonlinear equations. Some different propagation phenomena can also be produced through manipulating multi-soliton waves. The results in this paper might be helpful for understanding some physical phenomena described in plasmas.

Keywords coupled nonlocal nonlinear Schrödinger (CNNLS) equation; Darboux transformation; dark soliton; bright soliton

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1 Introduction

The Schrödinger equation is one of the basic equations of quantum mechanics proposed by physicist Schrödinger in 1926. In recently, Ablowitz and Musslimani [1,2] proposed some new nonlocal nonlinear integrable equations which include the

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nonlocal integrable nonlinear Schrödinger equation, mKdV equation, and so on. According to the relative scale of the relative length of the root beam width and the nonlinear response function of the medium, the nonlocal nonlinear Schrödinger equation can be divided into four classes [3] including the local class, weakly nonlocal class, general nonlocal class and strongly nonlocal class. The spatial solitons in nonlocal nonlinear media have attracted great interesting [4-13]. The research status of nonlocal spatial solitons were summarized and reviewed in [11].

With the further study of the soliton theory, it provides many methods for solving nonlinear partial differential equations, such as the homogeneous balance method [14], bilinear method [15], traveling wave method [15], Darboux transformation (DT) method [16], inverse scattering transform method [15-19]. For examples, some discrete rogue-wave solutions with dispersion in parity-time symmetric potential of Ablowitz-Musslimani equation were derived in [20]. Some bright, dark and breather wave soliton solutions of the super-integrable hierarchy were presented by Darboux transformation [21]. The non-autonomous multi-rogue wave solutions in a spin-1 coupled nonlinear Gross-Pitaevskii equation with varying dispersions, higher nonlinearities, gain/loss and external potentials were investigated in [22]. The generalized three-coupled Gross-Pitaevskii equations by means of the DT and Hirota's method were worked, and several non-autonomous matter-wave solitons including dark-dark-dark and bright-bright-bright shapes were obtained in [23]. The non-autonomous discrete vector bright-dark solutions and their controllable behaviors in the coupled Ablowitz-Ladik equation with variable coefficients were considered in [24]. The Darboux transformation method with 4×4 spectral problem are applied to study a specific equation and then the explicit solutions of the lattice integrable coupling equation were obtained in [25, 26].

The spectral problem stems from a solution of nonlinear partial differential equations, and the new solution was derived by Darboux transformation method [16]. The Darboux transformation can get a new solution from a known equation, also some multi-soliton solutions of the nonlinear partial differential equation can be obtained through multiple Darboux transformation [27-32]. The coupled nonlinear schrödinger equation, which describes a nonlinear diffusion regularity of two nonlinear wave propagation in the medium, not only is applied widely in the field of nonlinear optics, but also plays an important role in meteorology.

Wu and He generated the derivative nonlinear Schrödinger (NLS) equations, whose nonlocal extensions are from Lie algebra splittings and automorphisms in [33]. A chain of nonsingular localized-wave solutions was derived for a nonlocal NLS equation with the self-induced parity-time (PT)-symmetric potential through the N-th Darboux transformation by Li and Xu in [34]. Some rational soliton solutions were