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Determining a Piecewise Conductive Medium Body by a Single Far-Field Measurement

Xinlin Cao[1], Huaian Diao[2] and Hongyu Liu[3,*]

[1] *Johann Radon Institute for Computational and Applied Mathematics, Altenberger Strasse 69, 4040 Linz, Austria.*
[2] *School of Mathematics and Statistics, Northeast Normal University, Changchun, Jilin 130024, China.*
[3] *Department of Mathematics, City University of Hong Kong, Kowloon, Hong Kong, China.*

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Abstract. We are concerned with the inverse problem of recovering a conductive medium body. The conductive medium body arises in several applications of practical importance, including the modelling of an electromagnetic object coated with a thin layer of a highly conducting material and the magnetotellurics in geophysics. We consider the determination of the material parameters inside the body as well as on the conductive interface by the associated electromagnetic far-field measurement. Under the transverse-magnetic polarisation, we derive two novel unique identifiability results in determining a 2D piecewise conductive medium body associated with a polygonal-nest or a polygonal-cell geometry by a single active or passive far-field measurement.

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Key words: Electromagnetic scattering, conductive transmission condition, inverse problem, single far-field measurement, polygonal, corner singularity.

1 Introduction

1.1 Physical motivation and mathematical formulation

We are concerned with the time-harmonic electromagnetic wave scattering from a conductive medium body. The conductive medium body arises in several applications of practical importance, including the modelling of an electromagnetic object coated with

*Corresponding author. *Email addresses:* xlcao.math@foxmail.com (X. Cao), hadiao@nenu.edu.cn (H. Diao), hongyu.liuip@gmail.com, hongyliu@cityu.edu.hk (H. Liu)

a thin layer of a highly conducting material and the magnetotellurics in geophysics. In order to well motivate the current study, we next provide brief discussions on the aforementioned two specific applications and then introduce the mathematical formulation of the associated inverse problems.

In what follows, the optical properties of a medium are specified the electric permittivity ε, the magnetic permeability μ and the conductivity σ. Let Ω be a bounded Lipschitz domain in $\mathbb{R}^2$ with a connected complement $\mathbb{R}^2\backslash\overline{\Omega}$. In the subsequent study of the unique identifiability results, Ω is assumed to be a polygon having a polygonal-nest or polygonal-cell partition; see Definition 2.1 and Definition 2.2 for more rigorous descriptions. Consider a infinitely long cylinder-like medium body $D:=\Omega\times\mathbb{R}$ in $\mathbb{R}^3$ with the cross section being Ω along the x_3-axis for $\mathbf{x}=(x_j)_{j=1}^3\in D$. In what follows, with a bit abuse of notation, we shall also use $\mathbf{x}=(x_1,x_2)$ in the 2D case, which should be clear from the context. Let $\delta\in\mathbb{R}_+$ be sufficiently small and $\Omega_\delta:=\{\mathbf{x}+h\nu(\mathbf{x});\ \mathbf{x}\in\partial\Omega \text{ and } h\in(0,\delta)\}$, where $\nu\in\mathbb{S}^1$ signifies the exterior unit normal vector to $\partial\Omega$. Set $D_\delta=\Omega_\delta\times\mathbb{R}$ to denote a layer of thickness δ coated on the medium body D. The material configuration associated with the above medium structure is given as follows:

$$\varepsilon,\mu,\sigma=\varepsilon_1,\mu_0,\sigma_1 \ \text{ in } D;\ \ \varepsilon_2,\mu_0,\frac{\gamma}{\delta}\ \text{ in } D_\delta;\ \ \varepsilon_0,\mu_0,0\ \text{ in } \mathbb{R}^3\backslash\overline{(D\cup D_\delta)}, \tag{1.1}$$

where for simplicity, ε_j, $j=0,1,2$, μ_0,γ are all positive constants and σ_1 is a nonnegative constant. Consider a time-harmonic incidence:

$$\nabla\wedge\mathbf{E}^i-\mathrm{i}\omega\mu_0\mathbf{H}^i=0,\quad \nabla\wedge\mathbf{H}^i+\mathrm{i}\omega\varepsilon_0\mathbf{E}^i=0\quad \text{in } \mathbb{R}^3, \tag{1.2}$$

where $\mathrm{i}:=\sqrt{-1}$, $\mathbf{E}^i$ and $\mathbf{H}^i$ are respectively the electric and magnetic fields and $\omega\in\mathbb{R}_+$ is the angular frequency. The impingement of the incident field $(\mathbf{E}^i,\mathbf{H}^i)$ on the medium body described in (1.1) generates the electromagnetic scattering, which is governed by the following Maxwell system:

$$\begin{cases} \nabla\wedge\mathbf{E}-\mathrm{i}\omega\mu\mathbf{H}=0,\quad \nabla\wedge\mathbf{H}+\mathrm{i}\omega\varepsilon\mathbf{E}=\sigma\mathbf{E}, & \text{in } \mathbb{R}^3,\\ \mathbf{E}=\mathbf{E}^i+\mathbf{E}^s,\quad \mathbf{H}=\mathbf{H}^i+\mathbf{H}^s, & \text{in } \mathbb{R}^3,\\ \lim\limits_{r\to\infty} r(\mathbf{H}^s\wedge\hat{\mathbf{x}}-\mathbf{E}^s)=0, & r:=|\mathbf{x}|,\ \hat{\mathbf{x}}:=\mathbf{x}/|\mathbf{x}|, \end{cases} \tag{1.3}$$

where as usual one needs to impose the standard transmission conditions, namely the tangential components of the electric field $\mathbf{E}$ and the magnetic field $\mathbf{H}$ are continuous across the material interfaces ∂D and ∂D_δ. The last limit in (1.3) is known as the Silver-Müller radiation condition.

Under the transverse-magnetic (TM) polarisation, namely,

$$\mathbf{E}^i=\begin{bmatrix}0\\0\\u^i(x_1,x_2)\end{bmatrix},\quad \mathbf{H}^i=\begin{bmatrix}H_1(x_1,x_2)\\H_2(x_1,x_2)\\0\end{bmatrix},$$