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Phase-Field Modeling and Numerical Simulation for Ice Melting

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Abstract. In this paper, we propose a mathematical model and present numerical simulations for ice melting phenomena. The model is based on the phase-field modeling for the crystal growth. To model ice melting, we ignore anisotropy in the crystal growth model and introduce a new melting term. The numerical solution algorithm is a hybrid method which uses both the analytic and numerical solutions. We perform various computational experiments. The computational results confirm the accuracy and efficiency of the proposed method for ice melting.

AMS subject classifications: 65M06, 65D17, 68U05, 68U07, 93A30

Key words: Allen-Cahn equation, phase-field model, ice melting.

1. Introduction

Melting is an important problem which is associated to various engineering field such as electroslag melting, welding and thawing of moist soil. Melting is the process of heating a substance to change it from solid to liquid, which is a common type of state change. Heat transfer is a physical phenomenon in physics, which refers to the phenomenon of heat transfer caused by temperature difference. Some melting models of heat transfer have been proposed in the past decades [20, 31, 40, 41, 45]. In [24], the authors applied a melting model based on the enthalpy-porosity method

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to investigate the effects of porosity on the ice melting process and heat transfer. Experimental results for the characterisation of the freezing and melting processes for water contained in spherical ice thermal storage elements were described and evaluated [10]. Three-dimensional melting of ice around a liquid-carrying tube placed in an adiabatic rectangular cavity was investigated by numerical analysis [39]. Fujishiro and Aoki [13] presented volume modeling of the phenomenon by mathematical morphology and cellular automaton using voxels to represent ice objects and calculated the heat conduction and melting effects based on volume operation. Zheng [49] used a lattice Boltzmann method with an interfacial tracking method to solve melting problem in an enclosure. Jones presented a method for animating melting solids and proposed a method to simulate the melting process taking account of the thermal flow and the latent heat caused by the phase change [19]. For animating materials that melt, flow, and solidify, Carlson presented a fast and stable system and simulated the melting of solids such as waxes by treating solids as fluids with very high viscosities [5]. Melting and flowing behaviors were simulated by solving the Navier-Stokes equations. Although this method can simulate the melting and flowing of high viscosity materials, it is not applicable to simulations of ice melting because the viscosity of water is low. Paiva *et al.* proposed a physical simulation for melting viscoplastic objects [32].

In addition, some works presented the melting phenomena with a perspective of computational vision [12, 15, 26, 28, 38, 48]. In this study, we focus on ice melting by using a mathematical modeling. We propose a model to investigate the ice melting with the modified Allen-Cahn (AC) equation [1,8]. In the proposed model, the temperature field is added to model the phenomenon of heat transfer for ice melting. Furthermore, we analyze the physical phenomenon of the ice cubes with different shapes. The proposed model is based on the phase-field method. The most significant computational advantage of the phase-field method is that an explicit tracking of the interface is unnecessary [11]. In a sharp interface method, it is necessary to solve highly coupled equations to track the evolution of individual interfaces during transformation [33]. In the phase-field method, however, we can describe the evolution of the phase-field with relatively simple equations involving mass and heat changes. As the reverse process of ice melting, the phenomena of crystal growth have been widely simulated by using a phase-field model [3,7,18,34,43]. However, there is little investigation for ice melting such as the melting process from ice to water. Therefore, we propose a mathematical modeling and present numerical simulations for ice melting in this paper.

The contents of this paper are as follows. In Section 2, we present a phase-field model for ice melting based on the modified AC equation. In Section 3, we describe a robust hybrid numerical method for the proposed model. In Section 4, we perform numerical experiments. Finally, we conclude in Section 5.

2. The phase-field model

We propose a phase-field method [29, 35] for ice melting simulation. We introduce a phase-field $\phi(\mathbf{x}, t)$ whose value is close to 1 if $\mathbf{x}$ is in the ice and is close to -1 if $\mathbf{x}$