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Bifurcations and Exact Solutions of the Raman Soliton Model in Nanoscale Optical Waveguides with Metamaterials*

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Abstract In this paper, we study Raman soliton model in nanoscale optical waveguides with metamaterials, having polynomial law non-linearity. By using the bifurcation theory method of dynamic systems to the equations of $\phi(\xi)$, under 24 different parameter conditions, we obtain bifurcations of phase portraits and different traveling wave solutions including periodic solutions, homoclinic and heteroclinic solutions for planar dynamic systems of the Raman soliton model. Under different parameter conditions, 24 exact explicit parametric representations of the traveling wave solutions are derived. The dynamic behaviors of these traveling wave solutions are meaningful and helpful for us to understand the physical structures of the model.

Keywords Raman soliton model, Planar dynamic systems, Bifurcations of phase portraits, Traveling wave solutions.

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1. Introduction

With the development of communication technology, optical communication is considered to be the most promising communication method that transmits signals by using optical waves in optical waveguides. As early as 1973, A. Hasegawa and F. Tappert proposed the concept of "optical soliton", which is an optical pulse wave maintaining its amplitude, shape and speed after colliding with other similar solitons [5]. Meanwhile, it was proved theoretically that the optical soliton can be propagated, when the dispersion effect and the nonlinear self-phase modulation effect reach a balance in the fiber. Therefore, the propagation of stable optical solitons has become the focus of current research in nonlinear optics [5].

In practical applications, we need to consider the loss of optical pulse wave energy during signal transmission. Many optical metamaterials (MMS) with abundant optical properties are used as optical fibers, which have linear or nonlinear electromagnetic properties [14, 15]. Some MMS with the negative dielectric constant and magnetic permeability properties are called double negative material (DNG).

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Raman scattering can occur when the optical pulse signal transmits in these negative exponential DNG materials, which results in some modulation instability and affects the propagation of soliton. Raman soliton is the optical soliton pulse wave obtained by modulating Raman scattering effect and nonlinear effect in the transmission [13, 18].

Xiang [18] derived the propagation equation of Raman solitons in MMS by Maxwell equation, which is the dimensionless and nonlinear Schrodinger equation:

$$\begin{aligned} &iq_t + aq_{xx} + (c_1|q|^2 + c_2|q|^4 + c_3|q|^6)q \\ &= i\alpha q_x + i\lambda(|q|^2q)_x + i\nu(|q|^2)_x q + \theta_1(|q|^2q)_{xx} + \theta_2|q|^2q_{xx} + \theta_3 q^2 q^*_{xx}, \end{aligned} \tag{1.1}$$

where $a \neq 0$, $q(x,t)$ is the complex-valued wave function with the independent variables x and t(where x is the spatial variable, and t is the temporal variable). The first term represents the temporal evolution of nonlinear wave, while the coefficient a is the group velocity dispersion (GVD). The coefficients c_j for $j = 1, 2, 3$ correspond to the nonlinear terms. Meanwhile, they form polynomial law nonlinearity. It must be noted here that when $c_2 = c_3 = 0$ and $c_1 \neq 0$, the equation (1.1) collapses to the Kerr-law nonlinearity. However, if $c_3 = 0$, $c_1 \neq 0$ and $c_2 \neq 0$, one arrives at parabolic-law nonlinearity. Thus, polynomial law stands as an extension to Kerr-law and parabolic-law. On the right-hand side of (1.1), α represents the coefficient of inter-modal dispersion. This arises when the group velocity of light propagating through a metamaterial is dependent on the propagation mode in addition to chromatic dispersion. The factors λ and ν are accounted for self-steepening for preventing shock-waves and nonlinear dispersion. Finally, the terms with θ_j for $j = 1, 2, 3$ arise in the context of optical metamaterials.

A large class of solitons and ultrashort pulse propagation can be obtained by modulating the linear and nonlinear term coefficients in the propagation equation (1.1) [13, 18]. In 2014, By using the function variable method and first integral method, A. Biswas et al. [3] gave a small number of periodic wave solutions and soliton solutions including light and dark solitons when equation (1.1) has Kerr-law nonlinearity. Subsequently, in the paper [1] published in the same year, A. Biswas et al. used the experimental method to demonstrate the propagation of solitons in MMS, and found that the soliton energy dissipation was caused by the high loss of this double negative material. In addition, by the aid of ansatz method, they obtained some light and dark solitons of equation (1.1). Furthermore, by employing the simple equation method, they found some exact wave solutions of the equation (1.1) including some solitons and period solutions for the Kerr-law nonlinearity [2]. After years, E. V. Krishnan et al. used the mapping function method to drive some periodic wave solutions and solitary wave solutions of the equation (1.1) with Kerr-law nonlinearity and parabolic-law nonlinearity [6] respectively. These solutions obtained in [6] are richer than the solutions obtained by A. Biswas et al. in 2014. Later, M. Veljkovic et al. [17] gave the parameter conditions of ultrashort pulses and numerical simulations of solitons by using set variables. In 2015, Y. Xu and A. Biswas et al. [20] applied the traveling wave hypothesis to model (1.1) for the first time. They set that equation (1.1) has traveling wave solutions with Kerr-law and parabolic-law. Then, in 2016, by using the same method, Y. Xu et al. [19] gave the exact implicit solutions of equation (1.1) with the third kind of elliptic integral form, and made some numerical simulations. In [16], by employing the improved modified extended tanh-function method, the extended trial equation method, the extended Jacobi elliptic function expansion method and the exp (η)-