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On a Comparative Study on Some Trigonometric Classes of Distributions by the Analysis of Practical Data Sets

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Abstract The aim of this paper is twofold. Firstly, we elaborate and investigate a new trigonometric class of distribution, called the type II Tan-G class. Secondly, we perform a practical comparative evaluation of certain trigonometric classes; the Sin-G, Cos-G, Tan-G classes and the new one, with each other and with their common baseline distribution. More specifically, the usefulness and flexibility of these trigonometric classes are demonstrated through twelve practical data sets, by using the Weibull distribution as baseline. Among the data sets, two of them concern the Covid-19 pandemic in France from March to June 2020. As main results, it is shown that the related trigonometric models can outperform the former Weibull model in various cases and that the proposed type II Tan Weibull model can be, in certain situations, the best of them. The main lines of the code written in the R software language are provided.

Keywords Trigonometric classes of distributions, Weibull distribution, Data fitting, Covid-19 pandemic.

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1. Introduction

In recent years, there has been great interest in the general classes of trigonometric distributions. This interest lies in the fact that such classes are fairly simple to handle, with well documented and accessible mathematical properties, and they generally offer excellent fits of different types of practical data sets. In this paper, we bring an assessment to these trigonometric classes of distributions. Before going further, let us briefly present them. We begin with the most popular one: the Sin-G class of distributions introduced by [11] and [19] and completed by [20]. The starting point is an univariate baseline distribution, defined by a cumulative distribution function (cdf) denoted by $G(x)$, with a probability density function (pdf) denoted by $g(x)$. This baseline distribution is very general; it can be of any support and have no, one or more parameters. Then, the Sin-G class is defined by the cdf given as

$$F(x) = \sin\left[\frac{\pi}{2}G(x)\right], \quad x \in \mathbb{R},$$

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and possesses the pdf specified by

$$f(x) = \frac{\pi}{2} g(x) \cos\left[\frac{\pi}{2} G(x)\right], \quad x \in \mathbb{R}.$$

The qualities of the Sin-G class are the following ones: (i) It is based on simple functions, (ii) no new parameter is added; the Sin-G distribution only depends on the parameters involved in the baseline distribution, (iii) it satisfies the following first-order stochastic dominance: $F(x) \geq G(x)$, $x \in \mathbb{R}$, and (iv) the related models can be easily enriched for some baseline distributions. The qualities of the Sin-G class have motivated the constructions of other trigonometric classes. The two most famous of them are the Cos-G class of distributions provided by [19] and [21], and the Tan-G class given by [19], [22] and [2], which will be treated in our study.

In this paper, we are also going to introduce a new class of trigonometric distributions along side the more classics ones, called the type II Tan-G (TIIT-G) class. With the above notations, it is defined by the cdf given as

$$F(x) = 1 - \tan\left[\frac{\pi}{4}(1 - G(x))\right], \quad x \in \mathbb{R}, \tag{1.1}$$

the corresponding pdf being defined by

$$f(x) = \frac{\pi}{4} g(x) \frac{1}{\left\{\cos\left[\frac{\pi}{4}(1 - G(x))\right]\right\}^2}, \quad x \in \mathbb{R}. \tag{1.2}$$

The definition of $F(x)$ is based on the second type of the T-X transformation by [1] applied to the unit tangent cdf as generator; we can write it as

$$F(x) = 1 - F_o(1 - G(x)),$$

where $F_o(x) = \tan[(\pi/4)x]$. The first type of the T-X transformation applied to $F_o(x)$ providing the cdf of the former Tan-G class defined by $F_*(x) = F_o(G(x))$. With this remark in mind, we see that the structural relationship between the TIIT-G and Tan-G classes is the same as that between the Sin-G and Cos-G classes. The definition of the TIIT-G class is therefore legitimate in this regard. As special features, (i) the TIIT-G class is a sub-class of the original T-X class by [1] (as shown later), (ii) the TIIT-G class also corresponds to the class combining the M class by [12] and the Tan-G class (as shown later), and (iii) from the inequality $\tan(y) \leq (4/\pi)y$ for $y \in [0, \pi/4]$, we deduce the following first-order stochastic dominance: $F(x) \geq G(x) \geq F_*(x)$, $x \in \mathbb{R}$. Thus, the TIIT-G class can be considered as an alternative to the Tan-G distribution class, while retaining similar flexibility and simplicity. It offers a new modeling option, extending the scope of the existing trigonometric classes of distributions. In the paper, we develop this point by exploring various of its theoretical and applied aspects.

As main part, applications are made to expose the wide coverage of different practical data sets which can be well fitted by trigonometric classes of distributions, by adopting the Weibull distribution as the baseline. These applications also exhibit the usefulness of each of the above classes depending on the data set. Moreover, in regard to the difficult times we are in living with the Covid-19 threat, some of these applications are about this pandemic. More precisely, it concerns the Covid-19 pandemic in France from March to June 2020. The objective is to offer a model that fits as best possible, aiming to be better prepared in the future in the face of