Journal of Nonlinear Modeling and Analysis
Volume 3, Number 2, June 2021, 321–334
http://jnma.ca; http://jnma.ijournal.cn
DOI:10.12150/jnma.2021.321

Stability Analysis of a Diffusive Predator-prey Model with Hunting Cooperation*

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Abstract In this paper, we are concerned with the dynamics of a diffusive predator-prey model that incorporates the functional response concerning hunting cooperation. First, we investigate the stability of the semi-trivial steady state. Then, we investigate the influence of the diffusive rates on the stability of the positive constant steady state. It is shown that there exists diffusion-driven Turing instability when the diffusive rate of the predator is smaller than the critical value, which is dependent on the diffusive rate of the prey, and the semi-trivial steady state and the positive constant steady state are both locally asymptotically stable when the diffusive rate of the predator is larger than the critical value. Finally, the nonexistence of nonconstant steady states is discussed.

Keywords Predator-prey model, Hunting cooperation, Stability, Turing bifurcation.

MSC(2010) 35K57, 35B32, 92D25.

1. Introduction

In predator-prey models, functional response illustrates the relationship between prey and predator. Recent efforts have revealed that aggregated predators cooperate with each other in hunting. In [5], Conser *et al.* proposed the following functional response incorporating hunting cooperation

$$f(u(t), v(t)) = \frac{Ce_0uv}{1 + hCe_0uv}, \tag{1.1}$$

where $u(t)$, $v(t)$, C, e_0 and h are the population density of prey at time t, the population density of predator at time t, fraction of prey caught by a predator per encounter, total encounter rate between the two species and handing time per prey respectively, and C, e_0 and h are all positive. Other types of functional response concerning hunting cooperation were introduced in [1,2]. In [13], Ryu *et al.* mathematically investigated the predator-prey model proposed by Conser *et al.* [5], which

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*The authors were supported by Natural Science Foundation of Zhejiang Province of China (No. LY19A010010) and National Natural Science Foundation of China (Nos. 11971143, 12071105).

involves the functional response (1.1) and takes the nondimensional form

$$\begin{cases} \frac{du(t)}{dt} = u(1-u) - \frac{\alpha uv^2}{1+uv}, \\ \frac{dv(t)}{dt} = \frac{\beta uv^2}{1+uv} - \gamma v, \end{cases} \tag{1.2}$$

where $\alpha,\ \beta > 0$ illustrate interspecific effects between two species and $\gamma > 0$ stands for the death rate of predator. They analyzed the stability of constant nonnegative equilibria and demonstrated the existence of Hopf, saddle-node and Bogdanov-Takens bifurcations [13].

In fact, animals always move around in order to survive. To model the spatial distribution of animals, it is necessary to introduce diffusion into the population dynamics. Diffusive predator-prey models have been discussed by many mathematicians and have been suggested to exhibit complex dynamics, including Turing patterns [18, 19], nonconstant steady states [4, 11, 20], periodic solutions [20, 23] and travelling waves [6–8]. Recently, there has been a growing interest on diffusive predator-prey models with hunting cooperation. In particular, the predator-prey model with hunting cooperation proposed by Alves and Hilker [1] was extended to include diffusion in [3, 12, 14–17, 21]. Self-diffusion and homogeneous Neumann boundary conditions were considered in [3,15,21], where the existence and stability of nonnegative equilibria, definitive boundedness of solutions, Hopf bifurcation and Turing instability were studied and it was shown that spatial patterns occur only when the prey spreads faster than the predator. In [12], Ryu and Ko focused on self-diffusion and homogeneous Dirichlet boundary conditions and investigated the asymptotic behavior of positive solutions when hunting cooperation in predators is strong. In [17], Song *et al.* considered self-diffusion and Allee effect in prey and explored self-diffusion-driven Turing instability under the assumption that the prey spreads slower than the predator. In [16], the existence, stability and Hopf bifurcation of the positive equilibrium were explicitly determined for the original model proposed by Alves and Hilker [1] and the existence and stability of spatial patterns induced by cross-diffusion were theoretically analyzed under the assumption that the prey spreads slower than the predator. In [14], Singh and Banerjee incorporated hunting cooperation into Holling type II functional response and numerically investigated the self-diffusion-driven spatial patterns. On the other hand, Zhang and Zhu considered diffusive predator-prey models with predator interference or foraging facilitation proposed by Berec [2], and studied the dynamical behaviour and pattern formation in [24].

However, there are few studies on the diffusive version of model (1.2). In [22], cross diffusion was introduced into model (1.2) by Yan *et al.*, and it was shown that cross diffusion can give rise to Turing instability and complicated patterns. In this paper, we consider the random nature of diffusion and investigate the modification of model (1.2) that incorporates self-diffusion and homogeneous Neumann boundary