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Global Exponential Stability in Lagrange Sense for Delayed Memristive Neural Networks with Parameter Uncertainties

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Abstract This paper addresses the Lagrange stability of memristive neural networks with leakage delay and time-varying transmission delays as well as parameter uncertainties. Based on the theory of Filippov's solution, by using Lyapunov-Krasovskii functionals and the free-weighting matrix method, sufficient conditions in terms of linear matrix inequality (LMI) are given to ascertain the networks with different kinds of activation functions to be stable in Lagrange sense. Meanwhile the estimation of globally attractive sets are given. Finally, numerical simulations are carried out to illustrate the effectiveness of theoretical results.

Keywords Memristive neural networks, Lagrange stability, Leakage delay, Uncertain parameters.

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1. Introduction

In the past few years, there have been increasing research interests in analyzing the dynamical behaviors of neural networks due to their widespread applications in various areas, such as optimization, signal processing, pattern recognition and so on. In these applications, the stability of neural networks is a precondition to ensure the results to be reliable. Up to now, much work has been done in the filed of stability of neural networks [1–5]. Most of these researches are about Lyapunov stability of monostable neural networks with a unique equilibrium attracting all trajectories asymptotically. However, monostable neural networks have been found computationally restrictive in many applications and multistable neural networks may be more appropriate [6, 7]. For example, the neural networks are required to have multistable equilibria when designed for associative memory or pattern recognition, so that they can get different results with diverse inputs (or initial values). In these applications, the neural networks are no longer globally stable in Lyapunov sense and it's meaningful to analyse their stability in Lagrange sense. Lagrange stability is concerned with the boundedness and the attractivity of systems. It has been proved that no equilibrium, chaos attractor or periodic state exists outside the global attractive set in a Lagrange stable neural network [6, 8]. Moreover, the global stability in Lyapunov sense can be regarded as a special case of stability in

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Lagrange sense when the attractive set is an equilibrium. So far, some researches about Lagrange stability of neural networks can be found in [6, 8–14].

Since the first memristor was fabricated by Hewlett-Packard Laboratory [15,16], it has attracted much attention because of its special features. Electrical measurements showed that memristors can produce many important features of synapses, such as long term potentiation, long term depression, spike-timing-dependent plasticity, and short term potentiation. Also the distinctive ability to memorize the passed quantity of electric charge and the nonvolatile nature make memirstor a potential media for the next generation storage technology. Memristive neural networks are constructed by replacing the resistors with memristors in VLSI circuits of conventional neural networks. This new type of neural networks will provide the great potential for building a brain-like neural computer by implementing the synapses of biological brains [17]. In [18], Hu and Wang proposed a simplified mathematical model to characterize the pinched hysteretic feature of the memristor. A memristive neural networks model was given in this paper. The employing of memristors made the neural networks state-dependent switching. The state-dependent switching neural networks are discontinuous on the right-hand side, which implies they have more abundant dynamical behaviors and are more difficult to be investigated. Moreover, the analysis of dynamical properties for the memristive neural networks has been found useful to address a number of interesting engineering tasks [19].

As well known, delays are inherent features in many practical networks. Transmission delays in neural networks can be caused by the finite switching speed of the neuron amplifiers and the finite signal propagation speed. In [6, 8–10], this kind of delays were considered when the authors analysed the Lagarange stability of neural networks. In [20], Gopalsamy proposed a kind of delays called leakage delay. Leakage delays are introduced to describe that the decay process of neurons is not instantaneous and time is required to isolate the static state. They always have a great impact on the dynamical behavior of neural networks [19, 21]. In [11], the authors studied the Lagrange stability of complex-valued neural networks with leakage delay. However, few scholars considered parameter uncertainties in the study of Lagrange stability for neural networks. Parameter uncertainties may arise because of the variations in system parameters (temperature variation for example), modeling errors or some ignored factors. These uncertainties may cause the instability and poor performance [21]. So, it's important to study the dynamical behaviors of neural networks by taking the uncertainties into account. In [21] and [22], Xiao et. al and Song et. al investigated the passivity of conventional and memristive neural networks with parameter uncertainties, respectively. Comparing with researches on passivity of neural networks without parameter uncertainties in [23] and [24], their results are more general and reasonable.

To our best knowledge results on Lagrange stability of memristive neural networks with leakage delay and parameters uncertainties have not been reported in the literature. Compared with traditional neural networks, the dynamical properties of memristive neural networks are more complex and difficult to analyse. To obtain more general and applicable results, delays and parameter uncertainties should be considered in the analysis. These factors also complicate the dynamical behavior of memristive neural networks, especially the leakage delay. Meanwhile, the activation functions in neural networks are various, but most of them are Lipschitz continuous or bounded.