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Spatial Dynamics of a Diffusive Predator-prey Model with Leslie-Gower Functional Response and Strong Allee Effect

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Abstract In this paper, spatial dynamics of a diffusive predator-prey model with Leslie-Gower functional response and strong Allee effect is studied. Firstly, we obtain the critical condition of Hopf bifurcation and Turing bifurcation of the PDE model. Secondly, taking self-diffusion coefficient of the prey as bifurcation parameter, the amplitude equations are derived by using multi-scale analysis methods. Finally, numerical simulations are carried out to verify our theoretical results. The simulations show that with the decrease of self-diffusion coefficient of the prey, the preys present three pattern structures: spot pattern, mixed pattern, and stripe pattern. We also observe the transition from spot patterns to stripe patterns of the prey by changing the intrinsic growth rate of the predator. Our results reveal that both diffusion and the intrinsic growth rate play important roles in the spatial distribution of species.

Keywords Predator-prey model, Leslie-Gower functional response, Allee effect, Turing bifurcation, Amplitude equations, Pattern formation.

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1. Introduction

Lotka-Volterra equation is the classical model to study the interaction between prey and predator populations, in which the prey population is assumed to be of logistic growth in the absence of predator. It provides the basis for the study of predator-prey model later [1–3]. In actual nature, the resources and environment is constantly changing and the environmental capacity of the predator may has the relation to the number of prey. Leslie-Gower formula can be used to describe exactly this situation, which is introduced by Leslie et al. [4–6], and satisfies the following assumptions [5, 7, 8]:

- The reduction in the number of the predator and individual capture rate is correlated.
- The density of the predator is proportional to the adaptive capacity and the amount of prey.

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These assumptions emphasize that the growth rate of the predator and prey is not infinite [9], which is in accordance with the situation with the limited resources.

Meanwhile, the prey population is affected by its own density, mate, resources and other factors, which will inhibit or promote the growth of the prey population [10]. Allee effect is one of the most important factors in the study of biological population, which has attracted extensive attention due to its biological significance [11–13]. For instance, Voorn et al. [14] deal with the bifurcation analysis of two predator-prey models with strong Allee effect and it can cause the bistability. Wang et al. [15] studied global bifurcation analysis of a class of general predator-prey models with strong Allee effect in prey population and the results suggest that overexploitation could lead to the extinction of predator-prey populations. In a recent paper [16], Nicole et al. conducted complete qualitative studies of the following model:

$$\begin{cases} \dfrac{du}{dt} = ru(1-\dfrac{u}{K})(1-\dfrac{m+b}{u+b}) - quv, \\ \dfrac{dv}{dt} = sv(1-\dfrac{v}{nu}), \end{cases} \tag{1.1}$$

where $u(t)$ and $v(t)$ are respectively the densities of prey and predator populations; r and s denote respectively the intrinsic growth rates of prey and predator; K represents the environmental capacity of prey; nu represents the environmental carrying capacity of the predator, which characterize Lesile-Gower function response, and $A(u) = 1 - \frac{m+b}{u+b}$ represents the Allee effect [17–19]. Considering the small population extinction rate is higher, it is required that $b > 0$, $-b < m < K$, namely $m + b > 0$. Based on [16], model (1.1) is topologically equivalent to

$$\begin{cases} \dfrac{du}{dt} = ((1-u)(u-M) - Qv(u+B))u^2, \\ \dfrac{dv}{dt} = S(u-v)(u+B)v. \end{cases} \tag{1.2}$$

It is well known that spatial dynamics among predator-prey populations have been one of the main research topics in recent years [20]. Diffusion can also break the stability of equilibrium, which may lead to the generation of spatial pattern. In the primitive spatial ecosystem, the spatial distribution of predator-prey depends on the influence of geographical location, climate, season and other conditions, such as the free movement of species, predator environmental capacity, prey refuge effect and Allee effect. So, in this paper, we consider the effects of diffusion of two species on spatial dynamics of model (1.2). The model has the following form:

$$\begin{cases} \dfrac{\partial u(x,y,t)}{\partial t} = ((1-u)(u-M) - Qv(u+B))u^2 + d_{11}\nabla^2 u, & (x,y) \in \Omega, t > 0, \\ \dfrac{\partial v(x,y,t)}{\partial t} = S(u-v)(u+B)v + d_{22}\nabla^2 v, & (x,y) \in \Omega, t > 0, \\ \dfrac{\partial u}{\partial \mathbf{n}} = \dfrac{\partial v}{\partial \mathbf{n}} = 0, & (x,y) \in \partial\Omega, t > 0 \end{cases} \tag{1.3}$$

with initial conditions:

$$u(x,y,0) \geq 0, \ v(x,y,0) \geq 0,$$

where d_{11} and d_{22} respectively represent the self-diffusion coefficient of prey and predator; the boundary $\partial\Omega$ is smooth and Ω is a bounded region of $\mathbb{R}^N$; the outward