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Existence and Blowup of Solutions for Neutral Partial Integro-differential Equations with State-dependent Delay

Jianbo Zhu[1], Xingxing Wang[1] and Xianlong Fu[1,†]

Abstract In this paper, we study the existence and blowup of solutions for a neutral partial functional integro-differential equation with state-dependent delay in Banach space. The mild solutions are obtained by Sadovskii fixed point theorem under compactness condition for the resolvent operator, the theory of fractional power and α-norm are also used in the discussion since the nonlinear terms of the system involve spacial derivatives. The strong solutions are obtained under the lipschitz condition. In addition, based on the local existence result and a piecewise extended method, we achieve a blowup alternative result as well for the considered equation. Finally, an example is provided to illustrate the application of the obtained results.

Keywords Neutral partial integro-differential equation, Analytic semigroup, Resolvent operator, Fractional power operator, State-dependent delay.

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1. Introduction

In this paper, we study the existence and blowup of solutions for the semilinear neutral partial integro-differential equation with state-dependent delay of the form

$$\begin{cases} \dfrac{d}{dt}\left[x(t)+F(t,x_t)\right] = -A[x(t)+F(t,x_t)] + \displaystyle\int_0^t \Upsilon(t-s)x(s)ds + G(t,x_{\rho(t,x_t)}), \\ t\in[0,T], \\ x_0=\varphi\in\mathscr{B}_\alpha, \end{cases} \tag{1.1}$$

where $-A$ is the infinitesimal generator of an analytic semigroup on a Banach space X, $\Upsilon(t)$ is a closed linear operator defined later, F, G and ρ are given continuous functions to be specified below, and $\mathscr{B}_\alpha$ is an abstract phase space endowed with a seminorm $\|\cdot\|_{\mathscr{B}_\alpha}$.

Partial integro-differential equations can be used to describe a lot of natural phenomena arising from many fields such as fluid dynamics, biological models and

[†]the corresponding author.
Email address: zhujianbo789@163.com(J. Zhu), 13262990727@163.com(X. Wang), xlfu@math.ecnu.edu.cn(X. Fu)
[1]School of Mathematical Sciences, Shanghai Key Laboratory of PMMP, East China Normal University, Shanghai 200241, China
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chemical kinetics etc. These are often more accurate than the classical differential equations. A very effective approach to study this kind of equations is to transfer them into integro-differential evolution equations in abstract spaces. Grimmer etc [1–3] proved the existence of solutions of the following integrodifferential evolution equation

$$\begin{cases} v'(t) = Av(t) + \int_0^t \Upsilon(t-s)v(s)ds + g(t), & \text{for } t \geq 0, \\ v(0) = v_0 \in X, \end{cases} \tag{1.2}$$

where $g : \mathbb{R}^+ \to X$ is a continuous function. They obtained the representation of solutions, the existence and uniqueness of solutions via resolvent operator associated to the following linear homogeneous equation

$$\begin{cases} v'(t) = Av(t) + \int_0^t \Upsilon(t-s)v(s)ds, & \text{for } t \geq 0, \\ v(0) = v_0 \in X. \end{cases}$$

That is, the resolvent operator $R(t)$, replacing the role of C_0-semigroup for evolution equations, plays an important role in solving Eq. (1.2) in weak and strict senses. From then on, some topics for nonlinear integro-differential evolution equations, such as existence and regularity, stability, (asymptotic) periodicity of solutions and control problems, have been investigated by many mathematicians through applying the theory of resolvent operators, see [4–10]. Particularly, Lin and Liu [4] investigated existence, uniqueness and regularity of mild solutions for semilinear integrodifferential equations involving nonlocal initial conditions

$$\begin{cases} u'(t) = A\left[u(t) + \displaystyle\int_0^t F(t-s)u(s)ds\right] + f(t, u(t)), & t \in [0, T], \\ u(0) + g(t_1, \cdots, t_p, u(t_1), \cdots, u(t_p)) = u_0, \end{cases}$$

in a Banach space X with A the generator of a strongly continuous semigroup and $F(t)$ a bounded operator for $t \in [0, T]$. f is a C^1 function. In paper [5] the authors studied the following partial functional integro-differential equations with infinite delay in Banach space

$$\begin{cases} u'(t) = Au(t) + \int_0^t \Upsilon(t-s)u(s)ds + f(u_t), & \text{for } t \geq 0, \\ u_0 = \varphi \in \mathscr{B}. \end{cases} \tag{1.3}$$

Under the assumptions that $f : \mathscr{B} \to X$ is continuously differentiable, and f' is locally Lipschitz continuous, the local existence and regularity of mild solutions for Eq. (1.3) were obtained there.

Meanwhile, the nonlinear neutral functional integro-differential equations with time delay have also been investigated extensively in these years, see [11–18] and the references therein. Ezzinbi etc [11,12] studied the existence of mild solutions for this type of neutral equations by using Banach fixed point theorem, and the regularity of solutions was discussed there under the conditions that the nonlinear functions are continuously differentiable. On the other hand, functional (integro) differential equations with state-dependent delay appear frequently in various models and hence the study of this kind of equations has received great attention in the last years too. Some recent works can be found in [19–28]. Hernández etc [19] investigated