

The Dynamics of Stochastic Predator-prey Models with Non-constant Mortality Rate and General Nonlinear Functional Response*

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Abstract In this paper, we investigate the dynamics of stochastic predator-prey models with non-constant mortality rate and general nonlinear functional response. For the stochastic system, we firstly prove the existence of the global unique positive solution. Secondly, we establish sufficient conditions for the extinction and persistence in the mean of autonomous stochastic model and obtain a critical value between them. Then by constructing an appropriate Lyapunov function, we prove that there exists a unique stationary distribution and it has ergodicity in the case of persistence. Finally, numerical simulations are introduced to illustrate our theoretical results.

Keywords Stochastic predator-prey model, Non-constant mortality rate, General nonlinear functional response, Extinction, Stationary distribution.

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1. Introduction

The dynamics of the predator-prey system has always been one of the hot topics of ecological science research. In the past decades, a lot of predator-prey models have been proposed and used to describe the food supply relationship between two species [1]. In [2], Cavani and Farkas considered the following predator-prey system with Holling type-II functional response

$$\begin{cases} \dot{N}(t) = \varepsilon N(t) \left(1 - \frac{N(t)}{K}\right) - \frac{aP(t)N(t)}{\beta + N(t)}, \\ \dot{P}(t) = P(t) \left(-M(P(t)) + \frac{bN(t)}{\beta + N(t)}\right), \end{cases} \quad (1.1)$$

where $N(t)$ and $P(t)$ are the densities of the prey and the predator at time t , respectively. All the parameters are positive constants. ε represents specific growth rate of the prey without predation and environmental constraints; K denotes the carrying capacity of the prey in the absence of predators; a , b and β are satiation coefficients or conversion rates; and here the function $M(P)$ is the specific mortality rate of predators in the absence of prey. At the same time, $M(P)$ could be constant or non-constant. If $M(P)$ is a constant (such as $M(P) = n$), the model (1.1) is the

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classic predator-prey model with Holling type-II functional response. In this paper, the mortality rate of predators

$$M(P) = \frac{\gamma + \delta P}{1 + P} = \delta + \frac{\gamma - \delta}{1 + P}, \quad (0 < \gamma < \delta)$$

is non-constant and it depends on the quantity of predator; here, γ is the mortality at low density, and δ is the maximal mortality with the natural assumption $\delta > \gamma$. Compared with the common models, the advantage of this model is that the predator mortality rate is neither a constant nor an unbounded function, and it increases with the increase of the quantity of predator. There have been many papers published about the model and its deformations, see [3–5].

In population dynamics, one significant component of the relationship between the predator and the prey is the functional response of the predator, which refers to the change in the density of prey attached by per unit time per predator as the density of prey changes. There have been several well-known types of nonlinear functional response: Holling type-II, type-III [6], Hassell-Varley type [7], Beddington-DeAngelis type [8–10], Crowley-Martin type [11], ratio-dependence type [12], etc. These important nonlinear response functions allow us to gain insight into the dynamic relationship between predators and preys. Therefore, it is reasonable to use a nonlinear functional response when we establish a predator-prey model. Based on model (1.1), we consider the following predator-prey model with general nonlinear functional response

$$\begin{cases} \dot{N}(t) = \varepsilon N(t) \left(1 - \frac{N(t)}{K}\right) - a\varphi(N(t))P(t), \\ \dot{P}(t) = P(t) \left(-\frac{\gamma + \delta P(t)}{1 + P(t)} + b\varphi(N(t))\right), \end{cases} \quad (1.2)$$

where $\varphi(N)$ is a general functional response.

However, environmental noise is ubiquitous in nature and population models are inevitably affected by environmental noise, which is an important part of reality [13,14]. Deterministic models have certain limitations in the mathematical modeling process of ecological models. For example, they are not easy to fit data and not easy for us to accurately predict the future dynamics of the system. Therefore, it is necessary to consider stochastic fluctuations in the modeling process of the population model. In recent years, there have been many significant papers on the dynamics of stochastic population models [15,16]. However, few papers have considered stochastic predator-prey models with non constant mortality rate and general nonlinear functional response in stochastic environments. In this paper, we will study the dynamics of the following stochastic predator-prey model

$$\begin{cases} dN = \left(\varepsilon N(t) \left(1 - \frac{N(t)}{K}\right) - a\varphi(N(t))P(t)\right) dt + \sigma_1 N(t) dB_1(t), \\ dP = P(t) \left(-\frac{\gamma + \delta P(t)}{1 + P(t)} + b\varphi(N(t))\right) dt + \sigma_2 P(t) dB_2(t). \end{cases} \quad (1.3)$$

where $B_1(t)$, $B_2(t)$ are mutually independent standard Brownian motions defined on a complete probability space $(\Omega, \mathcal{F}, \mathcal{F}_{t \geq 0}, \mathcal{P})$ with a σ -field filtration $\{\mathcal{F}_t\}_{t \geq 0}$ satisfying the usual conditions, and σ_i^2 represent the intensities of the white noise, $i = 1, 2$.

For the sake of biological reality, we give the general assumption for $\varphi(N)$ above. Again, for the sake of clarity, we make two further assumptions for the generic nature