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Threshold Dynamics of a Time-periodic Reaction-Diffusion Malaria Model with Distributed Latencies*

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Abstract It is well-known that the transmission of malaria is caused by the bites of mosquitoes. Since the life habit of mosquitoes is influenced by seasonal factors such as temperature, humidity and rainfall, the transmission of malaria presents clear seasonable changes. In this paper, in order to take into account the incubation periods in humans and mosquitoes, we study the threshold dynamics of two periodic reaction-diffusion malaria models with distributed delay in terms of the basic reproduction number. Firstly, the basic reproduction number R_0 is introduced by virtue of the next generation operator method and the Poincaré mapping of a linear system. Secondly, the threshold dynamics is established in terms of R_0. It is proved that if $R_0 < 1$, then the disease-free periodic solution of the model is globally asymptotically stable; and if $R_0 > 1$, then the disease is persistent.

Keywords Incubation period, the basic reproduction number, periodic solution, distributed latency, uniform persistence.

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1. Introduction

Malaria is a serious infectious disease with a long history of development. It is widely believed in biology that malaria originates in Africa from a parasite commonly known as Plasmodium, which was first found in chimpanzees. Malaria is spread widely among the population through female adult mosquitoes and poses a great threat to human health. According to the latest WHO malaria report, there are more than 200 million people suffering from malaria in the world. The data shows that 90% of malaria cases occur in African countries. In addition, India is also the main country of malaria infection. According to statistics, the number of cases in malaria-prone countries increased by nearly 3.6 million in 2018, with 40% of the deaths due to the illness. In China, malaria often occurs in Sichuan, Yunnan and

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Guangxi. Due to the different sources of infection, temperature, humidity and so on, it is very difficult to control and eradicate malaria. Based on the latest news, there are about 2600 cases of malaria in China, and the proportion of deaths has increased. This shows that malaria still has a huge hidden danger to human life and there is a long way to go to achieve the goal of malaria-free.

Mathematical model is a basic and effective tool for studying the mechanism of disease transmission. A reasonable mathematical model reflects the law of disease development and predict its changing trend. It can provide good suggestions and guides for people to prevent,control and eradicate disease. Therefore, many scientists begin to study the dynamics of disease transmission through mathematical model.

The first mathematical model for malaria transmission was introduced by Ross [37] in 1911. Ross proposed a system of ordinary differential equations which studys the malaria transmission between humans and mosquitoes. It proved that the prevalence of diseases would be controlled when the number of mosquitoes was less than the threshold value. Subsequent contributions have been made by Macdonald [32, 33] to the generalization of the classical Ross-Macdonald model, that is,

$$\begin{cases} \frac{dh(t)}{dt} = ab\frac{H-h(t)}{H}v(t) - rh(t), \\ \frac{dv(t)}{dt} = ac\frac{h(t)}{H}(V - v(t)) - dv(t). \end{cases}$$

Here H and V are the total populations of humans and mosquitoes, respectively. $h(t)$ and $v(t)$ are the numbers of infected humans and mosquitoes at time t, a is the rate of biting on humans by a single mosquito, b and c are the transmission probabilities from infected mosquitoes to susceptible humans and from infected humans to susceptible mosquitoes, respectively, $\frac{1}{r}$ is the duration of the disease in humans and d is the mortality rate of mosquitoes.

Macdonald obtained several interesting conclusions through researching the Ross-Macdonald model. Firstly, the result states that malaria can persist in a population only if the number of mosquitoes is greater than a given threshold. Secondly, the prevalence of infection in the human and the mosquito hosts depends directly on the basic reproduction number and the relationship is nonlinear. Thirdly, the model has a stable positive equilibrium when the basic reproduction number is greater than 1. This means that temporary intervention can lead to a temporary reduction of prevalence, when the intervention is relaxed prevalence again increased to the original values. Moreover, Macdonald performed a sensitivity analysis of the basic reproduction number. He found that halving the mosquito population reduces R_0 by a factor of two, meanwhile halving biting rate reduces R_0 by a factor of four. The largest reduction of R_0 is expected for increase in adult mosquito mortality. The work of Macdonald had a very beneficial impact on the collection, analysis, and interpretation of epidemic data on malaria infection and guided the enormous global malaria-eradication campaign of his era.

However, the Ross-Macdonald model ignores many ecological and epidemiological factors, such as the age structure, acquired immunity in humans, spatial heterogeneity, temperature, climate and latency and so on. But these factors play a great influence on the dynamics of malaria transmission. So a number of researchers begin to focus on this important aspect by including these factors. For example, Forouzannia and Gumel [14] take into account the age structure. An important conclusion is that the disease-free equilibrium is locally asymptotically stable if $R_0 < 1$,